Chapter 1 Units and Dimensions

Units and measurements: What are CGS AND SI system of units? Mention of fundamental quantities in CGS and SI.

Dimensions Of Physical Quantities: What is meant by dimension of a Physical quantity? Writing the dimensions of important fundamental quantities in mechanics: Mass, Length, Time.

Dimensional Formula: What is dimensional formula? Writing the dimensional formula of various derived physical quantities using its defining formula. **Examples:** Velocity, Momentum, Acceleration, Force, Work, Energy, Torque, Angular Momentum, Moment of Inertia, Stress, Time period, Frequency, Pressure, Surface Tension, Coefficient of Viscosity.

Dimensional formula of some constants: Speed of light, Universal Gravitational Constant.

Errors In Physics: Observed and true value, Accuracy and Precision, Significant figures with examples. Types of Errors: Systematic and Random Errors with examples. Definitions of Independent and dependent errors, Absolute error and Percentage error. Estimation of errors: Mention of formulae to estimate for dependent and independent errors, Average or mean value in measurements. Definition of Standard deviation and expression for standard deviation. Numerical Problems

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CGS system: The **centimetre (C), gram (G), second (S)** system of units is a variant of the metric system, based on the **centimetre** as the unit of length, the **gram** as the unit of mass, and the **second** as the unit of time.

SI system: The **International System of Units**, known by the **SI system. I**t is the only system of measurement employed in science, technology, industry and commerce.

The SI comprises a system of units of measurement starting with **seven base units**, namely, the **second** (s, time), **metre** (m, length), **kilogram** (kg, mass), **ampere** (A, electric current), **kelvin** (K, absolute temperature), **mole** (mol., amount of substance) and **candela** (cd, luminous intensity).

There are two more supplementary quantities, namely, **Plane angle** (radian) and **Solid angle** (steradian).

Fundamental quantities: The quantities that are independent of other quantities are called **fundamental quantities**. The units that are used to measure these fundamental quantities are called **fundamental units**.

Examples: length (m), mass (kg), time (s) etc.

Derived quantities: The quantities that are derived using the fundamental quantities are called **derived quantities**. The units that are used to measure these derived quantities are called **derived units**.

Examples: velocity (m/s), acceleration (m/s^2), force (mass x acceleration) etc.

Dimensions and Dimensional Formula:

Dimensions of the physical quantity are the power to which, the base quantities are raised to represent that quantity. **Examples:** i) area = side x side = L^2. Here, 2 is the dimension.

ii) volume = side x side x side = L^3. Here, 3 is the dimension.

Dimensional formula: Dimensional formula of any physical quantity is an expression, which represents how and which of the base quantities are included in that quantity. It is written by enclosing the symbols for base quantities with appropriate power in **square brackets**.

Examples: Dimension formula of length is [L], mass is [M], time is [T], area is [L^2], etc.

Dimensional equation: The equation obtained by equating a physical quantity with its dimensional formula is called a **dimensional equation**.

Dimensional Constants: The physical quantities which have dimensions and have a fixed value are called dimensional constants. **Examples:** Gravitational constant (G), Planck's constant (h), Universal gas constant (R), Velocity of light in a vacuum (c), etc.

Dimensionless quantities: Dimensionless quantities are those, which do not have dimensions but have a fixed value.

i) Dimensionless quantities without units: Pure numbers, π, e, sinθ, cosθ, tanθ etc.
ii) Dimensionless quantities with units: Angular displacement (radian), Joule's constant (joule/calorie), etc.

Dimensional variables: Dimensional variables are those physical quantities which have dimensions and do not have a fixed value. **Examples:** velocity, acceleration, force, work, power, etc.

Dimensionless variables: Dimensionless variables are those physical quantities which do not have dimensions and do not have a fixed value. **Examples:** Specific gravity, refractive index, coefficient of friction, Poisson's ratio, etc.

Limitations of Dimensional Analysis:

i) Dimensionless quantities cannot be determined by this method. Constant of proportionality cannot be determined by this method.
ii) This method is not applicable to trigonometric, logarithmic and exponential functions.
iii) In the case of physical quantities, which are dependent upon more than three physical quantities, this method will be difficult.
iv) In some cases, the constant of proportionality also possesses dimensions. In such cases, we cannot use this system.
v) If one side of the equation contains addition or subtraction of physical quantities, we cannot use this method to derive the expression.

Dimensional Formulae for Physical Quantities:

Physics quantity	Defining Formula	SI unit	Dimensional formula
Area	$Length\ (L) \times length\ (L)$	m^2	**[L^2]**
Volume	$L \times L \times L$	m^3	**[L^3]**
Density	$mass/volume$	kg m^3	**[M L^3]**
speed	$distance/time\ taken$	ms^{-1}	**[L T^{-1}]**
velocity	$displacement$ $/time\ taken$	ms^{-1}	**[L T^{-1}]**
Acceleration	$velocity\ /\ time\ taken$	ms^{-2}	**[L T^{-2}]**

Momentum	$Mass \times velocity$	kg ms^{-1}	$[M L T^{-1}]$
Force	$Mass \times acceleration$	N (or) kg m s^{-2}	$[M L T^{-2}]$
Moment of a force (moment of a couple)	$Force \times distance$	N.m	$[M L^2 T^{-2}]$
Work done	$Force \times displacement$	N.m (or) joule	$[M L^2 T^{-2}]$
Energy	$Capacity\ to\ do\ work$	joule	$[M L^2 T^{-2}]$
Impulse of a force	$Force \times time$	N s (or) kg ms^{-1}	$[M L^2 T^{-1}]$
Moment of Inertia	$Mass \times (radius)^2$	Kg m^2	$[M L^2]$
Power	$Work/time$	Watt (or) J s^{-1}	$[M L^2 T^{-3}]$
Pressure	$Force/area$	Pascal (or) Nm^{-2}	$[M L^{-1} T^{-2}]$
Surface tension	$Force/unit\ length$	N m^{-1}	$[M T^{-2}]$
Frequency	$1/time\ period$	Hertz (or) per s	$[T^{-1}]$
Torque	$Force \times distance$	N m	$[M L^2 T^{-2}]$
Linear density	$mass/unit\ length$	kg m^{-1}	$[M L^{-1}]$
Aerial velocity	$Area/time$	$m^2\ s^{-1}$	$[L^2 T^{-1}]$
Stress	$Restoring\ force/area$	Pascal (or) N m^{-2}	$[M L^{-1} T^{-2}]$
Strain (a ratio)	$\frac{change\ in\ dimension}{original\ dimension}$	Ratio has no unit	No dimensional formula
Modulus of elasticity	$stress/strain$	Pascal (or) N m^{-2}	$[M L^{-1} T^{-2}]$
Coefficient of Viscosity	$\frac{Force \times radius}{araea \times velocity}$	Pascal. second	$[M L^{-1} T^{-1}]$
Speed of light	$distance/time\ taken$	ms^{-1}	$[L T^{-1}]$
Universal Gas Constant	$R = \frac{pressure \times volume}{temperature}$	$J\ mol.^{-1} K^{-1}$	$[M L^2 T^{-2} K^{-1}]$
Universal Gravitational Constant	$G = \frac{Force \times (radius)^2}{(mass)^2}$	$N\ m^{-1} Kg^{-2}$	$[M^{-1} L^3 T^{-2}]$

Errors: The result of every measurement using any measuring instrument contains some uncertainty. This uncertainty is called **error**.

Accuracy and Precision: The **accuracy** of a measurement is a measure of how close the measured value is to the true value of the quantity. **Precision** gives to what limit or resolution the quantity is measured.

Accuracy may depend on several factors including the limit or resolution of the measuring instrument.
Example: Suppose the **true value** of a certain length is near 3.678 cm.

i) In an experiment (using a resolution of 0.1 cm), the measured value is found to be 3.5 cm.
ii) In an another experiment (using a resolution of 0.01 cm), the measured value is found to be 3.38 cm.

The first measured value has more accuracy as it is closer to the true value, but with less precision.

Types of errors: There are **three types of errors**, namely, systematic errors, random errors and least count errors.

1. **Systematic errors:** These are errors that tend to be in one direction, either positive or negative.

i) **Instrumental errors:** This is due to imperfection in design or calibration of the measuring instrument. **Example:** A thermometer, which reads 101°C for boiling water, which is an error, zero error in a screw gauge etc.

ii) **Imperfection in experimental procedure or technique**: **Example:** Temperature of the body should not be measured under the armpit, which gives lower than the actual body temperature.

iii) **Personal errors:** This occurs due to an individual's idea, lack of proper setting etc. **Example:** Parallax error.

2. **Random errors:** These are errors which occur irregularly, due to sign and size. These may arise due to random and unpredictable fluctuations. **Example:** Electrical experiments.
3. **Least count errors:** The smallest possible value that can be measured by the measuring instrument is called its **least count**. The least count error is the error associated with the resolution of the instrument. **Example:** A Vernier callipers has the least count of 0.01 cm. and a Screw gauge has the least count of 0.001 mm.

Note: Repeating the observations several times and taking the arithmetic mean, the error is minimized.

Absolute error: The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement. It is denoted by $|\Delta a|$.

If a_1, a_2, a_3, …., a_n are the measurements made with an instrument and the arithmetic mean,

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n} = \frac{\sum x_i}{n}$$

Then,

$$\Delta a_1 = a_1 - a_{mean}$$
$$\Delta a_2 = a_2 - a_{mean}$$
$$\Delta a_3 = a_3 - a_{mean}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\Delta a_n = a_n - a_{mean}$$

Note: Δa may be positive or negative, but the absolute error $|\Delta a|$ is always positive.

Relative error: Relative error is the ratio of the mean of absolute errors to the mean value of the quantity measured, i.e.,

$$\text{relative error} = \frac{|\Delta a_{mean}|}{a_{mean}}$$

Percentage error: When the relative error is represented in percentage, it is called percentage error.

$$\text{percentage error} = \frac{|\Delta a_{mean}|}{a_{mean}} \times 100$$