Unit-2

Laws of Motion

Newton's First Law :

Statement : "A body remains in a state of rest or of uniform motion unless acted uponby an external force." Newton's first law is also referred to as the **law of inertia**.

Inertia is defined as " *the tendency of an object to remain at rest or move with a constant velocity."* Inertia is a fundamental property of all matter and is depend upon the mass of the object. Mass is the quantity which is the measure of inertia, if mass of the object is more, inertia of that object has more. To change the state of rest or state of uniform motion of a body, an external force is necessary.

This implies that a force is necessary to change the state of rest or of uniform motion of a body.

Examples : If you kicked a ball in space, it would keep going forever.

Newton's Second law :

Statement : " The net force on an object is equal to product of the mass of the object and the acceleration of that particle." (**F** = m**a**).

Or

" the net force on a particle is equal to the time rate of change of its linear momentum."

Explanation : Mathematically, it is given by $F = \frac{dP}{dt}$

where, $P = mv$, is known as linear momentum of the particle.

hence, $F = \frac{d(mv)}{dt}$

Since, mass 'm' of the particle is constant,

therefore, $F = m\frac{dv}{dt}$

w.k.t $\frac{dv}{dt} = a$ (acceleration of the particle)

hence, $\boldsymbol{F} = \boldsymbol{ma}$

If $F = 0$, then from above equation, $0 = m\frac{dv}{dt}$. hence, $v = constant$. This verifies Newton's first law of motion.

Significance : Newton's Second Law says that the net force, **F**, acting on an object causes the object to accelerate. Since **F** = **ma** can be rewritten as **a** = **F/m**, you can see that the magnitude of the acceleration is directly proportional to the net force and inversely proportional to the mass, m. Both force and acceleration are vector quantities,and the acceleration of an object will always be in the same direction as the net force.

Examples: If you use the same force to push a truck and push a car, the car will have more acceleration than the truck, because the car has less mass.

Newton's Third law:

Statement: " To every action there is an equal and opposite reaction."

Explanation: If body A exerts a force F_{12} on body B, simultaneously, body B exerts a force F_{21} of the same magnitude on body A, both forces acting along the same line.

The two forces in Newton's third law are equal and opposite.

i.e $F_{12} = - F_{21}$

$$\frac{dP_1}{dt} = -\frac{dP_2}{dt}$$

$$\frac{d(mv_1)}{dt} = -\frac{d(mv_2)}{dt}$$

$$\therefore v_1 = -v_2$$

Hence linear momentum of the object is conserved.

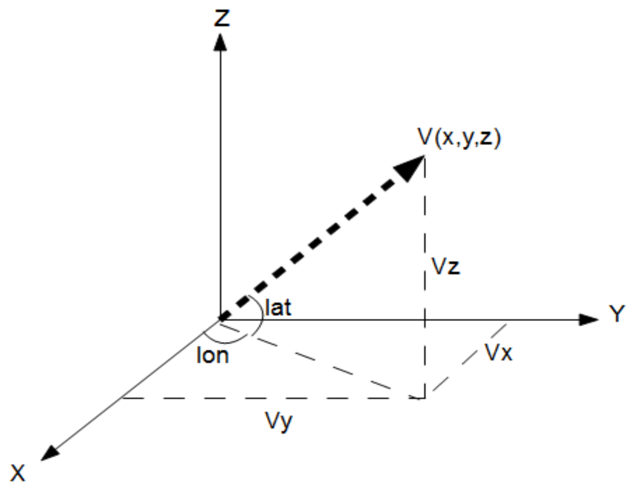
Examples: 1. When you jump off a small rowing boat into water, you will push yourself forward towards the water. The same force you used to push forward will make the boatmove backwards.

2. When you dive off of a diving board, you push down on the springboard. The board springs back and forces you into the air.

Limitations of Newton's law of motion :

1. Newton's law are valid only in inertial frame of reference. Since according to second law, mass of the object is constant, but mass of the moving object is not same, it varies w.r.t velocity of the object. When velocity of the object equals to speed of the light, mass of the object becomes infinity.
2. Newton's law of motion fail to explain the behavior and interaction of objects having atomic and molecular sizes.
3. It is impossible to simultaneous measurement of two equal and opposite forces for ordinary practical purposes.

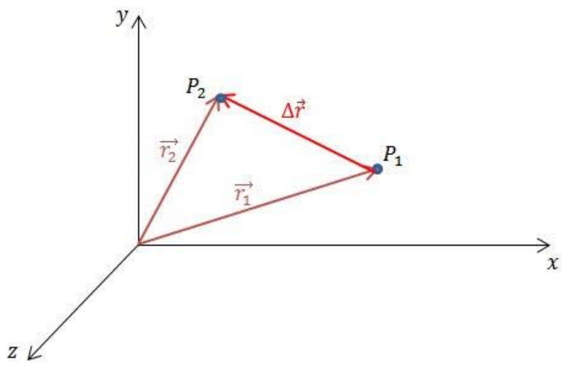
Position vector : It is a vector which represents the position or location of a point in a surface or a space.



In the above diagram, position vector of a particle is given by,

$$\vec{r} = V_x\hat{\imath} + V_y\hat{\jmath} + V_z\hat{k}$$

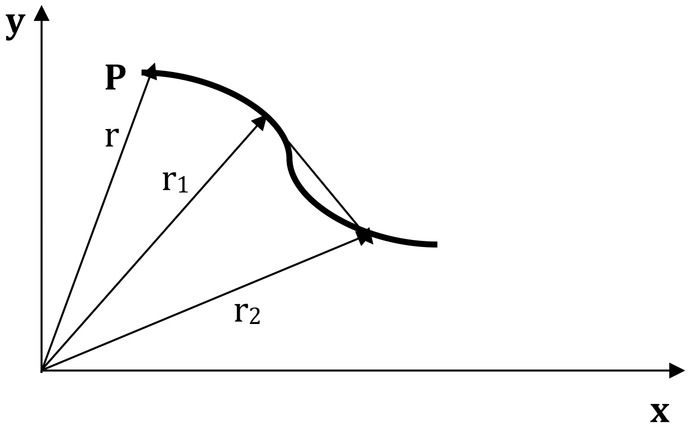
Displacement vector : It is a vector which represents distance between initial and final position of a particle in a plane or space.



In the above diagram, P_1 is the initial position of the particle and $\vec{r_1}$ is its position vector, while, P_2 is the final position of the particle and $\vec{r_2}$ is its position vector.
Then displacement vector $\Delta\vec{r}$ between P_1 and P_2 is given by
By Pythagoras theorem, $\vec{r_2} + \Delta\vec{r} = \vec{r_1}$

$$\therefore \quad \Delta\vec{r} = \vec{r_1} - \vec{r_2}$$

Dynamics of a Single particle:



Consider a point particle P is at rest with position vector $\vec{r}$. When a force is acts on a particle it moves in a curvilinear path as shown in the figure. When a particle is at initial position A, its position vector is $\vec{r_1}$ and moves to final position B, its position vector is $\vec{r_2}$.

Instantaneous velocity of the particle is given by, $V = \frac{d\vec{r}}{dt}$

Linear momentum of the particle is, $\vec{P} = m\vec{V}$

$$\text{Or} \quad \vec{P} = m\frac{d\vec{r}}{dt}$$

From Newton's II^{nd} law of motion, $\vec{F} = \frac{d\vec{P}}{dt}$

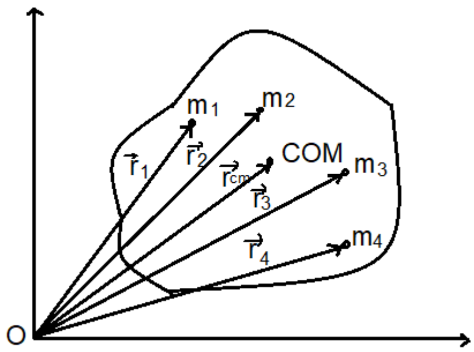
$$\text{Or} \quad \vec{F} = \frac{d(mv)}{dt} = m\frac{d}{dt}\left(\frac{d(\vec{r})}{dt}\right)$$

When F=0, velocity of the particle becomes constant. Hence, linear momentum of particle is constant. Therefore, linear momentum is conserved. And also $\frac{d^2r}{dt^2} = 0$, Therefore, displacement of the particle is zero. i.e. particle remains at rest.

Dynamics of System of Particles :

Centre of mass: *A point where if the entire mass of the system is to be concentrated is known as Centre of mass of that system.*

Consider a system of mass 'M' consisting of a 'N' number of particles of masses m_1, m_2, m_3..............m_N whose position vectors are $\overrightarrow{r_1}, \overrightarrow{r_2}, \overrightarrow{r_3} \ldots \ldots \ldots .. \overrightarrow{r_N}$ as shown in below diagram.



Let consider an i^{th} particle, whose mass is m_i and its position vector is $\overrightarrow{r_i}$. When force F is applied on the system, the force acting on i^{th} is,

$$\overrightarrow{F_i} = \frac{dP_i}{dt} = \frac{d(m_i \overrightarrow{v_i})}{dt} = \frac{d(m_i \frac{\overrightarrow{dr_i}}{dt})}{dt} = \frac{d^2(m_i \overrightarrow{r_i})}{dt^2}$$

For 'N' particles,

$$\sum_{i=1}^{N} \overrightarrow{F_i} = \sum_{i=1}^{N} \frac{d^2(m_i \overrightarrow{r_i})}{dt^2} = \frac{d^2 \Sigma(m_i \overrightarrow{r_i})}{dt^2}$$

$$\vec{F} = M \frac{d^2}{dt^2} \sum_{i=1}^{N} \frac{m_i \vec{r}_i}{M}$$

$$\vec{F} = M \frac{d^2 \vec{R}}{dt^2} \qquad \text{------------------ (1)}$$

Where, the position vector of Centre of mass is given by,

$$\vec{R} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2} + m_3 \overrightarrow{r_3} + \cdots \ldots . + m_N \overrightarrow{r_N}}{m_1 + m_2 + m_3 + \cdots \ldots \ldots . + m_N}$$

$$\vec{R} = \frac{\Sigma m_i \overrightarrow{r_i}}{\Sigma m_i} = \sum_{i=1}^{N} \frac{m_i \vec{r}_i}{M} = \frac{\Sigma m_i \overrightarrow{r_i}}{M} \qquad \text{--------------------- (2)}$$

Where, M is the mass of the system.

$\mathrm{M}\vec{R} = \sum_{i=1}^{N} m_i \vec{r}_i$

Differentiating w.r.t 't', we get,

$\mathrm{M}\,\frac{d\vec{R}}{dt} = \sum_{i=1}^{N} m_i \frac{dr_i}{dt} = \sum_{i=1}^{N} m_i \vec{v}_i$

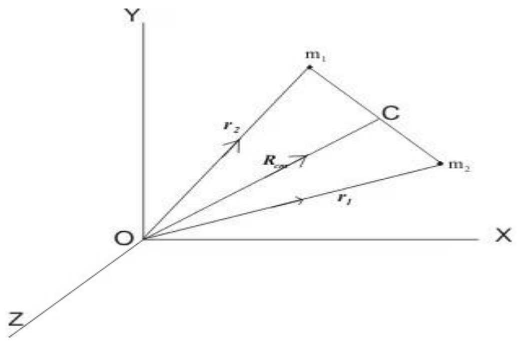
$\mathrm{M}\,\overrightarrow{v_{cm}} = \vec{P}$ ----------------- (3)

From eq(1), $\vec{F} = M\,\frac{d}{dt}\left[\frac{d\vec{R}}{dt}\right]$

$\vec{F} = M\,\frac{d}{dt}\left[\overrightarrow{v_{cm}}\right]$

If F=0, then $\overrightarrow{v_{cm}} = constant$. According to eq(3), linear momentum of the system becomes constant, hence linear momentum of the system is conserved.

For 2- particle System :



From eq (2), $\vec{R} = \sum_{i=1}^{N} \frac{m_i \vec{r}_i}{M} = \frac{\sum_1^N m_i \vec{r}_i}{\sum_1^N m_i}$

For 2 particles m_1 and m_2 , Position vector of Centre of mass becomes

$$\vec{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$