Linear Programming

1) A Person wants to decide the constituents amount of diet which will fulfill his daily requirements of Protiens,Fats and Carbohydrates at a Minimum cost. The choice is to be made from four different types of foods. The yields per unit of those food are given below. Formulate Linear programming Model

Food type	**Yield per unit**			**Cost per unit (Rs)**
	Proteins	**Fats**	**Carbohydrates**	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum Requirement	**800**	**200**	**700**	

Solution:

Let X_1 Amount of Type1 food to be taken, Let X_2 Amount of Type2 to be taken, Let X_3 Amount of Type3 to be taken, Let X_4 Amount of Type4 to be taken,

$Z_{min} = 45X_1 + 40X_2 + 85X_3 + 65X_4$ ----Objective Equation

$3X_1 + 4X_2 + 8X_3 + 6X_4 >= 800$ [Protein constrain]

$2X_1 + 2X_2 + 7X_3 + 5X_4 >= 200$ [Fats constrain]

$6X_1 + 4X_2 + 7X_3 + 4X_4 >= 700$ [Carbohydrates constrain]

$X_1>=0, X_2>=0, X_3>=0, X_4>=0$ { Non negative Constrain}

Linear Programming

1) Old Hens can be bought for Rs. 300 each , Young one cost Rs. 500 each, Old Hen lay 3 eggs per week and young ones lay 5 eggs per week, Each egg being worth 6 Rs. A hen costs of Rs. 15 for feed. If a person has only 15000 to spend on the hens, How many of each kind should he buy to get a profit of more than 600 Rs. Per week Assuming that he cannot house more than 30 hens.Formulate Linear programming Model

Solution:

Let X be the number of Old Hens to be brought,

Let Y be the number of Young Hens to be brought

	Old Hen	Young Hen
Cost	300	500
Egg	3 Egg per week	5 Egg per week
Feed	15 Rs.	15 Rs.
Each Egg Selling price	6	6

Z_{Max} = SP-CP=

=(6*3)X+(6*5)Y -15*X-15*Y {15* X indicate feed}

=18X+30Y-15X-15Y

Z_{Max}=3X+15Y ----Objective Equation

300X+500Y <= 15000 [Cost constrain]

X+Y <= 30 [Space constrain]

X>=0,Y>=0 [Non Negative]

3X+15Y >=600 [Income Constrain]

Linear Programming

1) A farmer has a 100 acre farm, He can sell all the Tomatoes,Onion, Radishes he can raise, The price he can obtain is Rs.5 per kg Tomatoes, Rs.10 per kg Onion,Rs. 8 per kg for Radish,The average yield per acre is 2000 kg of Tomatoes,3000 kg Onion,1000 kg of Radish, Fertilizer is available at RS.5 per Kg, Amount of fertilizer require for each Tomatoe,Onion-100 Kg per acre,50 Kg for Radish,Labour require for cultivating per ace is 5 man day for Tomatoes,6 man day for Onion,5 man day for Radish. A total 400 mans are available at Rs.500 per day. Formulate Linear programming Model.

Solution:

	Tomatoes	Onion	Radish
Selling Price	Rs.5 per kg	Rs.10 per kg	Rs. 8 per kg
Yield	2000kg/Acre	3000kg/Acre	1000kg/Acre
Fertilizer	100kg/Acre	100kg/Acre	50kg/Acre
Cultivating	5 Man/Acre	20 Man/Acre	5 Man/Acre

Fertilizer rate: 5 Rs/kg

Each Labour Cost: Rs.500 per Day

X_1	X_2	X_3

Let X_1 be the number of acres to be used for growing Tomatoe.

Let X_2 be the numbe of acrs to be used for growing Onion.

Let X_3 be the numbe of acrs to be used for growing Radishes.

Z_{max} =(2000*5) X_1 + (3000*10) X_2 + (1000*8) X_3 – (100*5) X_1 -(100*5) X_2 – (50*5) X_3 –(500*5) X_1 –(20*500) X_2- (500*5) X_3

$=10000X_1 + 30000\ X_2 + 8000\ X_3 - 500\ X_1 - 500\ X_2 - 250\ X_3 - 2500\ X_1 - 10000\ X_2 - 2500\ X_3$

$Z_{max} = 7000\ X_1 + 19500\ X_2 + 5250\ X_3$ { Objective equation }

<u>Constrains:</u>

$5\ X_1 + 20\ X_2 + 5\ X_3 <= 400$ {Labour Constrain}

$X_1 + X_2 + X_3 <= 100$ { Land Constrain}

$X_1 >= 0,\ X_2 >= 0,\ X_3 >= 0$ { Non negative Constrain }

2) A hospital has the following minimal daily requirement of nurses

Period	Clock time	Minimal Number of Nurses Requaired
1	6 AM to 10 AM	2
2	10 AM to 2 PM	7
3	2 PM to 6 PM	15
4	6 PM to 10 PM	8
5	10 PM to 2 AM	20
6	2 AM to 6 AM	6

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there is sufficient number of nurses available for each period. Formulate this as Linear programming problem

Solution:

Let $X_1, X_2, X_3, X_4, X_5, X_6$ number of nurses to be appointed for period 1,2,3,4,5,6 respectiveley.

Objective equation: $Z_{min} = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

Constrains

$X_1 + X_2 >= 7$

$X_2 + X_3 >= 15$

$X_3 + X_4 >= 8$

$X_4 + X_5 >= 20$

$X_5 + X_6 >= 6$

$X_6 + X_1 >= 2$

$X_1 >= 0, X_2 >= 0, X_3 >= 0, X_4 >= 0, X_5 >= 0, X_6 >= 0,$

Linear Programming

1) Consider the following problem faced by a production planner in a soft drink plant. He has 2 bottling machines A and B. A is assigned for 8-ounce bottle and B is assigned for 16-ounce bottle. The following data are available

Machine A	8ounce bottle	16ounce bottle
A	100/ Minute	40/Minute
B	60/Minute	75/Minute

The machines can be run for 8-hours per day,5 days a week, Profit on 8 ounce bottle is 15 paise and on 16 ounce bottle is 25 paise. Weekly production of the drink cannot exceed 300000 ounces and the market can obserb 25000 eight ounce bottle and 7000 sixteen ouce bottles per week. The planner wishes to maximize his profit,of course,to all the production and marketing constrains. Formulate Linear programming model.

Solution

Let X_1 be the number of 8 Ounce bottle to be produced on Machine A, X_2 be the number of 16 ounce bottle to be produced on Machine A.
Let Y_1 be the number of 8 ounce bottle to be produced on B, Y_2 be the number of 16 ounce bottle to be produced on B.

$Z_{max} = (X_1 + Y_1)\ 0.15 + (X_2 + Y_2)0.25$ [Objective equation]

1 Min----100 products then 1 product $\frac{1}{100}$ duration requires

Similarley for X_1 products $\frac{X_1}{100}$ [8 Ounce bottles]

1 Min----- 40 products 1 product $\frac{1}{40}$ duration requires

Similarley for X_2 products $\frac{X_2}{40}$ [16 Ounce bottles]

$$\frac{X_1}{100} + \frac{X_2}{40} <= 5*8*60 \qquad \frac{Y_1}{60} + \frac{Y_2}{75} <= 5*8*60$$

$$\frac{2X_1 + 5X_2}{200} <= 2400$$

$$2X_1 + 5X_2 <= 480000$$

$X_1 + Y_1 > = 25000$[Market Constrain 8 ounce bottle]
$X_2 + Y_2 > = 7000$ [Market Constrain 16 ounce bottle]

$8(X_1 + Y_1) + 16(X_2 + Y_2) <= 300000$

$X_1 >= 0, X_2 >= 0, Y_1 >= 0, Y_2 >= 0$

2) A manufacture of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredient available to make 20,000 bottles of A and 40000 bottles of B but there are only 45000 bottles into which both the medicines can be put. Further more it take 3 hours to prepareenough materials to fill 1000 bottles of A, it take 1 hour to prepare enough materials to fill 1000 bottles of B and there are 66 hours available for this operations. The profit is Rs.8 perbottle for A and Rs.7 perbottle for B. Formulate the linear programming model

Solution: Let x be the number of A type medicine to be produced
Let y be the number of B type medicine to be produced

$Z_{max} = 8x + 7y$ [objective]

x <=20000 [Bottle A constrain]
y<=40000[Bottle B constrain]
x+y<=45000[Together]

$$\frac{3}{1000} * x = \frac{3x}{1000}$$

$$\frac{1}{1000} * y = \frac{y}{1000}$$

$$\frac{3x}{1000} + \frac{3y}{1000} <= 66 \text{ hrs}$$ Hours constrain

X>=0, Y>=0

Linear Programming

Q1) A firm can produce 3 types of cloth A,B,C. 3 kinds of wool required for it, Red wool,green wool, and blue wool.

One unith length of type A cloth needs 2 yards of red Wool and 3 yards of blue wool.one unit of length of type B cloth need 3 yard of red wool, 2 yards of green wool and 2 yards of blue wool. One unit of type c cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has a stock of only 8 yards of red wool, 10 yads of green wool,and 15 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs 3, of type B cloth is Rs.5, of type C cloth is Rs. 4. Formulate the problem as a linear programming problem

Solution:

Let X_1 be the number of Type A cloth to be produced

Let X2 be the number of Type B cloth to be produced

Let X3 be the number of Type C cloth to be produced

$Z_{max} = 3X_1 + 5X_2 + 4X_3$ ----Objective Equation

	Red	Green	Blue
Type A	2 Yard	----	3 Yard
Type B	3 Yard	2 Yard	2 Yard
Type C	-----	5 Yard	4 Yard

Availability 8 Yard 10 Yard 15 Yard

$2X_1 + 3X_2 < = 8$ {Red Wool Constrain}

$2X_1 + 5X_3 <= 10$ {Green Wool Constrin}

$3X_1 + 2X_2 + 4X_3 <= 15$ { Blue Wool Constrain}

$X_1 >= 0, X_2 >= 0, X_3 >= 0$ { Non negative Constrain}

Q2) A firm Produce 3 Products,these products are processed in 3 different machine. The time required to manufacture one unit of each product and the daily capacity of machine 3 machine given below

Machine	**P1**	**P2**	**P3**	**Machine capacity**
M1	2	8	2	940 Min/day
M2	4	---	8	970 Min/day
M3	2	5	--	430 Min/day

Profit of product P1,P2,P3 are RS 4, Rs 8, Rs 6 Write a linear programming model to the given problem

<u>Solution:</u>

Let a be the number of Type P1 Product to be produced

Let b be the number of Type P2Product to be produced

Let c be the number of Type P3 Product to be produced

Z_{max} = 4a+ 8b+6c ----Objective Equation

2a+8b+2c <=940 { Machine P1 constrain}

4a+8c <=970 { Machine P2 constrain}

2a+5b <=430 { Machine P3 constrain}

a>=0, b>=0, c>=0 { Non negative Constrain}

Q3) A farmer has 1000 acres of land on which he grown corn,wheat, and soyabeans. Each acre of corn cost Rs. 100 for preparation, requires 7 man days of work and yields a profit of Rs 30. An acre of wheat costs Rs 120 to prepare, requires 10 man day of work and yields profit of Rs.40, An acre of soyabean costs Rs 70 to prepare requires 8 man days of work and yields of profit Rs.20, If the farmer has Rs. 100000 for preparation and can count on 8000 man days work, formulate the L.P model to allocate the number of acres to each crop to maximize the total profit

Solution:

Let X_1 be the land to be used to cultivating Corn

Let X2 be the land to be used to cultivating Wheat

Let X3 be the land to be used to cultivating Soyabean

$Z_{max} = 30X_1 + 40X_2 + 20X_3$ ----Objective Equation

	Preparation cost	Labour
Corn	100 RS. per Acre	7 Man days per Acre
Wheat	120 Rs. per Acre	10 Man days per Acre
Soyabean	70 RS.per Acre	80Man days per Acre

Availability 100000 RS 8000 Mans Available

Acres available: 1000

$100X_1 + 120X_2 + 70X_3 <= 100000$ { Preparation cost constrain}

$7X_1 + 10X_2 + 8X_3 <= 8000$ {Labour constrain}

$X_1 + X_2 + X_3 <= 1000$ { Acres constrain}

$X_1 >= 0, X_2 >= 0, X_3 >= 0$ { Non negative Constrain}

Linear Programming

Q1) A firm can produce 3 types of cloth A,B,C. 3 kinds of wool required for it, Red wool,green wool, and blue wool.

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Solution:

Let X_1 be the number of Type A cloth to be produced

Let X2 be the number of Type B cloth to be produced

Let X3 be the number of Type C cloth to be produced

$Z_{max} = 3X_1 + 5X_2 + 4X_3$ ----Objective Equation

	Red	Green	Blue
Type A	2 Yard	----	3 Yard
Type B	3 Yard	2 Yard	2 Yard
Type C	-----	5 Yard	4 Yard
Availability	**8 Yard**	**10 Yard**	**15 Yard**

$2X_1 + 3X_2 <= 8$ {Red Wool Constrain}

$2X_1 + 5X_3 <= 10$ {Green Wool Constrin}

$3X_1 + 2X_2 + 4X_3 <= 15$ { Blue Wool Constrain}

$X_1 >= 0, X_2 >= 0, X_3 >= 0$ { Non negative Constrain}

Q2) A firm Produce 3 Products,these products are processed in 3 different machine. The time required to manufacture one unit of each product and the daily capacity of machine 3 machine given below

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Profit of product P1,P2,P3 are RS 4, Rs 8, Rs 6 Write a linear programming model to the given problem

Solution:

Let a be the number of Type P1 Product to be produced

Let b be the number of Type P2Product to be produced

Let c be the number of Type P3 Product to be produced

Z_{max} = 4a+ 8b+6c ----Objective Equation

2a+8b+2c <=940 { Machine P1 constrain}

4a+8c <=970 { Machine P2 constrain}

2a+5b <=430 { Machine P3 constrain}

a>=0, b>=0, c>=0 { Non negative Constrain}

Q3) A farmer has 1000 acres of land on which he grown corn,wheat, and soyabeans. Each acre of corn cost Rs. 100 for preparation, requires 7 man days of work and yields a profit of Rs 30. An acre of wheat costs Rs 120 to prepare, requires 10 man day of work and yields profit of Rs.40, An acre of soyabean costs Rs 70 to prepare requires 8 man days of work and yields of profit Rs.20, If the farmer has Rs. 100000 for preparation and can count on 8000 man days work, formulate the L.P model to allocate the number of acres to each crop to maximize the total profit

Solution:

Let X_1 be the land to be used to cultivating Corn

Let $X2$ be the land to be used to cultivating Wheat

Let $X3$ be the land to be used to cultivating Soyabean

$\mathbf{Z_{max}} = 30X_1 + 40X_2 + 20X_3$ ----Objective Equation

	Preparation cost	Labour
Corn	100 RS. per Acre	7 Man days per Acre
Wheat	120 Rs. per Acre	10 Man days per Acre
Soyabean	70 RS.per Acre	80Man days per Acre

Availability	**100000 RS**	**8000 Mans Available**

Acres available: 1000

$100X_1 + 120X_2 + 70X_3 <= 100000$ { Preparation cost constrain}

$7X_1 + 10X_2 + 8X_3 <= 8000$ {Labour constrain}

$X_1 + X_2 + X_3 <= 1000$ { Acres constrain}

$X_1 >= 0, X_2 >= 0, X_3 >= 0$ { Non negative Constrain}

LinearProgramming

1) A computer manufacturing company purchases components parts and make 2models of moniters A and B. The components are assembled by the company to produce model A and B. Model A requires 28 hours of labour to assemble from component part, while model B requires 42 hours. After assembly each monitor is tested in inspection department. Model A requires 12 hours of inspection time while B requires 6 hours. The company employs 400 peoples in the assembly department, each working 7 hours a day, 6 days a week. 100 peoples are presently employed in the inspection department, each working 8 hours a day, 6 days a week. Currently wages rate are Rs. 50 per hour in assembly and Rs.75 per hour in inspection. Model A cost 1850 and Model B cost Rs.3250 to produce. Currently two models sell for Rs. 6400 and Rs. 8300. The supplier of these chip can provide no more than 660 in any one working week

Solution:

	Monitor A	**Monitor B**
Component Cost	1850	3250
Assemble Cost	1400	2100
Inspection Cost	900	450
Cost Price	4150	5800
Selling price	6400	8300
Profit	6400-4150= 2250	8300-5800=2500

Let X be the number of type A to be produced,

Let Y be the number of type B to be produced

Z_{max} = 2250 X + 2500 Y [Objective equation]

Assemble for per monitor A type requires 28 hour labour, 1 hour labour cost for assemble =50

for 28 hours =28* 50 =1400

Type B requires 42 hour labour for assemble

For 42 hours= 42*50 =2100

Inspection cost per hour=75

Type A—12= 12*75=900

Type B---- 6=6*75= 450

	Monitor A	Monitor B	Employees	working	total day	Total hour
Assemble	28 hours	42hours	400	7 hour/day	6	400*7*6= 16800
Inspection	12 hours	6 hours	100	8 hour/day	8	100*8*6= 4800

One Monitor Type A require 28 hour for assemble, So X monitor requires X* 28=28 X

One Monitor Type B require 42 hour for assemble, So Y monitor requires Y*42= 42 Y

Available hour in assemble department =16800 **28 X +42Y <=16800 [Assemble constrain]**

Similarly

One Monitor Type A require 12 hour for inspection, So X monitor requires X* 12=12 X

One Monitor Type B require 6 hour for assemble, So Y monitor requires Y*6=6 Y

Available hour in assemble department =4800 **12X+6Y<=4800**[**Inspection dept. hour constrain**]

Chip available 660

Assume that 1 monitor require 1 chip of any type X number of Type A require X number of chip, Y number of type B requires Y number of chip

X+Y<=660 [Chip Constarin] X>=0,Y>=0 [Non negative Constrain]

2) A confectionery company mixes 3 types of toffees **ingredients** to form one kg of toffee packs. The pack is sold at Rs. 170, The 3 types of toffees **ingredients** cost Rs.200, Rs.100 and Rs.50 per kg. The mixture must contain at least 0.3kg of the first type of toffees and the weight of first first two types of toffees **almost** be equal to the weight of third type. Determine the optimal mix for maximum profit

Solution : Let X, Y,Z be the number of KG ingredients of 3 types is mixed to prepare 1 KG toffees

Z_{max} = 170- 200 *X-100*Y- 50*Z [Objective equation]

X>=0.30 [First type ingredients constrain]

X+Y<=Z [type1 and 2 almost equal to third type]

X+Y+Z=1 [To add all that to form only 1 kg, not exceed or less than 1]

X>=0, Y>=0, Z>=0

Simplex Method

Q1) $Z_{Max} = 3X_1 + 4X_2$

$X_1 + X_2 <= 4$

$X_1 - X_2 <= 2$

Solution: $Z_{max} - 3X_1 - 4X_2 = 0$

$X_1 + X_2 + S_1 = 4$

$X_1 - X_2 + S_2 = 2$

Basic Variable: Unique, Unit, Positive Co-efficient : S_1, S_2, Z_{Max}

Basic variable	X_1	X_2	S_1	S_2	Z_{Max}	RHS	Ratio= $\frac{RHS}{PC}$
S_1	1	1	1	0	0	4	$\frac{4}{1}$ =4
S_2	1	-1	0	1	0	2	$\frac{2}{-1} = -2$
Z_{Max}	-3	-4	0	0	1	0	

Pivoting Column [P C] = Entering variable [E V] = Maximum Negative in Z_{Max} $x_2 = -4$

Pivoting Row [P R]= Leaving Variable [LV]= Minimum Positive Ratio = 4

Pivoting Element =Intersecting of Row and Column= 1

New Pivoting Row= $X_2 = \frac{1}{PE} * \text{Old Row}$

$X_2 = \frac{1}{1}$ *[1, 1, 1, 0, 0, 4]

$X_2 =$ [1, 1, 1, 0, 0, 4]

Key Element for $S_2 = -1$

New Row S_2 = - Key * NPR + Old row

= -(-1) (1, 1, 1, 0, 0, 4) + (1, -1, 0, 1, 0, 2)

= 1 (1, 1, 1, 0, 0, 4) + (1, -1, 0, 1, 0, 2)

= (1, 1, 1, 0, 0, 4) + (1, -1, 0, 1, 0, 2)

= (1+1, 1-1, 1+0, 0+1, 0+0, 4+2)

= (2 0 1 1 0 6)

Simplex Method

Key Element for Z_{Max} = -4

Z_{Max} = - Key * NPR + Old row

$= -(-4)\left(1, 1, 1, 0, 0, 4\right) + \left(-3, -4, 0, 0, 1, 0\right)$

$= 4 \left(1, 1, 1, 0, 0, 4\right) + \left(-3, -4, 0, 0, 1, 0\right)$

$= \left(4, 4, 4, 0, 0, 16\right) + \left(-3, -4, 0, 0, 1, 0\right)$

$= \left(4-3, 4-4, 4+0, 0+0, 0+1, 16+0\right)$

$= \left(1 \quad 0 \quad 4 \quad 0 \quad 1 \quad 16\right)$

Basic variable	X_1	X_2	S_1	S_2	Z_{Max}	RHS	Ratio= $\frac{RHS}{PC}$
X_2	1	1	1	0	0	4	
S_2	2	0	1	1	0	6	
Z_{Max}	1	0	4	0	1	16	

Stop the Process because there is no negative in Z_{max} row.

$1X_2 = 4$ $\quad 1Z_{Max} = 16$

$\boxed{X_2 = 4}$ $\quad \boxed{Z_{Max} = 16}$

There is no X_1 is present in Basic Variable Column So $X_1 = 0$

Verification:

$X_1 + X_2 <= 4$

$0 + 4 <= 4$

$4 <= 4$

$X_1 - X_2 <= 2$

$0-4 <= 2$

$-4 <= 2$

$Z_{Max} = 3X_1 + 4X_2$

$= 3(0) + 4(4)$

$= 0 + 16$

$Z_{Max} = 16$

Simplex Method

Q4) $Z_{min} = X_1 - 3X_2 + 2X_3$

$3X_1 - X_2 + 3X_3 <= 7$

$4X_1 + 3X_2 + 8X_3 <= 10$

$-2X_1 + 4X_2 <= 12$

Solution : $3X_1 - X_2 + 3X_3 + S_1 = 7$

$4X_1 + 3X_2 + 8X_3 + S_2 = 10$

$-2X_1 + 4X_2 + S_3 = 12$

Convert Z_{min} to Z_{max}

I.e. $Z_{max} = -Z_{min}$

$Z_{max} = -[X_1 - 3X_2 + 2X_3]$

$Z_{max} = -X_1 + 3x_2 - 2X_3$

$Z_{max} + X_1 - 3X_2 + 2X_3 = 0$

I. B.F.S.

Let $X_1 = 0$, $X_2 = 0$, $X_3 = 0$, then $S_1 = 7$, $S_2 = 10$, $S_3 = 12$

Basic variable :Unique, Unit, $+^{ve}$ Co-effcient

S_1, S_2, S_3, Z_{max}

B.V	X_1	X_2	X_3	S_1	S_2	S_3		Z_{max}	RHS	Ratio=RHS/P.C
S_1	3	-1	3	1	0	0		0	7	7/-1=-7
S_2	4	3	8	0	1	0		0	10	10/3
S_3	-2	4	0	0	0	1		0	12	3
Z_{max}	1	-3	2	0	0	0		1	0	

Simplex Method

Pivoting Element [P.E] = Intersting of row and column = 4

New Pivoting row = $\frac{1}{PE}$* [Old row]

$$=\frac{1}{4}\left[-2, 4, 0, 0, 0, 1, 0, 12\right]$$

$$=\left[-\frac{1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3\right]$$

Key element for S_1 = -1

New row S_1 = -Key *NPR+ Old row

$$= -(-1)\left[\frac{-1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3\right] + \left[3, -1, 3, 1, 0, 0, 0, 7\right]$$

$$= 1\left[\frac{-1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3\right] + \left[3, -1, 3, 1, 0, 0, 0, 7\right]$$

$$= \left[-\frac{1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3\right] + \left[3, -1, 3, 1, 0, 0, 0, 7\right]$$

$$= \left[-\frac{1}{2}+3, 1-1, 0+3, 0+1, 0+0, \frac{1}{4}+0, 0+0, 3+7\right]$$

$$= \left[\frac{5}{2}, \quad 0 \quad 3 \quad 1 \quad 0 \quad \frac{1}{4} \quad 0 \quad 10\right]$$

Key element for S_2 = 3

New row = S_2 = -Key *NPR +old row

$$= -3\left[-\frac{1}{2}, 1, 0, 0, 0, \frac{1}{4} 0, 3\right] + \left[4, 3, 8, 0, 1, 0, 0, 10\right]$$

$$=\left[\frac{3}{2}, -3, 0, 0, 0, -\frac{3}{4}, 0, -9,\right] + \left[4, 3, 8, 0, 1, 0, 0,10\right]$$

Simplex Method

$$= \left(\frac{3}{2} +4, -3+3, 0+8, 0+0, 0+1, -\frac{3}{4}+0, 0+0, -9+10 \right)$$

$$S_2 = \left(\frac{11}{2}, \quad 0, \quad 8, \quad 0, \quad 1, \quad -\frac{3}{4}, \quad 0, \quad 1 \right)$$

Key Element of Z_{max} = -3

New row Z_{max} = - Key * NPR +Old row

$$= -(-3) \left(\frac{-1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3 \right) + \left(1, -3, 2, 0, 0, 0, 1, 0 \right)$$

$$= \quad 3 \left(\frac{-1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3 \right) + \left(1, -3, 2, 0, 0, 0, 1, 0 \right)$$

$$= \quad \left(-\frac{3}{2}, 3, 0, 0, 0, \frac{3}{4}, 0, 9 \right) + \left(1, -3, 2, 0, 0, 0, 1, 0 \right)$$

$$= \quad \left(-\frac{3}{2}+1, 3-3, 0+2, 0+0, 0+0, \frac{3}{4}+0, 0+1, 9+0 \right)$$

$$= \quad \left(-\frac{1}{2}, \quad 0 \quad 2 \quad 0 \quad 0 \quad \frac{3}{4} \quad 1 \quad 9 \right)$$

B.V	X_1	X_2	X_3	S_1	S_2	S_3	Z_{max}	RHS	Ratio
S_1	$\frac{5}{2}$	0	3	1	0	$\frac{1}{4}$	0	10	$\frac{10}{\frac{5}{2}} = \frac{10*2}{5} = 4$
S_2	$\frac{11}{2}$	0	8	0	1	$-\frac{3}{4}$	0	1	$\frac{1}{\frac{2}{11}} = \frac{1*11}{2} = \frac{11}{2}$
X_2	$-\frac{1}{2}$	1	0	0	0	$\frac{1}{4}$	0	3	$\frac{3}{-\frac{1}{2}} = -6$
Z_{max}	$-\frac{1}{2}$	0	2	0	0	$\frac{3}{4}$	1	9	

Simplex Method

Pivoting Element [P.E] = Intersting of row and column = $\frac{11}{2}$

New Pivoting row = X_1 = $\frac{1}{PE}$ [Old row]

$$= \frac{1}{\frac{11}{2}} \left(\frac{11}{2}, 0, 8, 0, 1, \frac{-3}{4}, 0, 1 \right)$$

$$= \frac{2}{11} \left(\frac{11}{2}, 0, 8, 0, 1, \frac{-3}{4}, 0, 1 \right)$$

$$NPR = X_1 = \left(1, 0, \frac{16}{11}, 0, \frac{2}{11}, \frac{-3}{22}, 0, \frac{2}{11} \right)$$

Key Element of $X_2 = -\frac{1}{2}$

New row X_2 = - Key * NPR +Old row

$$= \left(1, 0, \frac{16}{11}, 0, \frac{2}{11}, -\frac{3}{22}, 0, \frac{2}{11} \right) + \left(-\frac{1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3 \right)$$

$$= \frac{1}{2} \left(1, 0, \frac{16}{11}, 0, \frac{2}{11}, -\frac{3}{22}, 0, \frac{2}{11} \right) + \left(-\frac{1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3 \right)$$

$$= \left(\frac{1}{2}, 0, \frac{8}{11}, 0, \frac{1}{11}, -\frac{3}{44}, 0, \frac{1}{11} \right) + \left(-\frac{1}{2}, 1, 0, 0, 0, \frac{1}{4}, 0, 3 \right)$$

$$= \left(\frac{1}{2} - \frac{1}{2}, 0+1, \frac{8}{11}+0, 0+0, \frac{1}{11}+0, -\frac{3}{44}+\frac{1}{4}, 0+0, \frac{1}{11}+3 \right)$$

$$= \left(0 \quad 1 \quad \frac{8}{11} \quad 0 \quad \frac{1}{11} \quad \frac{8}{44} \quad 0 \quad \frac{34}{11} \right)$$

Simplex Method

B.V	X_1	X_2	X_3	S_1	S_2	S_3	Z_{max}	RHS	Ratio
S_1	0	0	$-\frac{7}{11}$	1	$-\frac{5}{11}$	$\frac{26}{44}$	0	$\frac{105}{11}$	
X_1	1	0	$\frac{16}{11}$	0	$\frac{2}{11}$	$-\frac{3}{22}$	0	$\frac{2}{11}$	
X_2	0	1	$\frac{8}{11}$	0	$\frac{1}{11}$	$\frac{8}{44}$	0	$\frac{34}{11}$	
Z_{max}	0	0	$\frac{30}{11}$	0	$\frac{1}{11}$	$\frac{30}{44}$	1	$\frac{100}{11}$	

$X_1 = \frac{2}{11}$ $\quad X_2 = \frac{34}{11}$ $\quad Z_{max} = \frac{100}{11}$

Simplex Method

Q2) Solve : $Z_{max} = x_1 + 2x_2$

S.T $-x_1+2x_2 <= 8$

$x_1-2x_2 <= 3$

$x_1+2x_2 <= 12$

Solution: $-x_1+2x_2+s_1=8$

$x_1-2x_2+s_2=3$

$x_1+2x_2+s_3=12$

$Z_{max} - x_1-2x_2= 0$

Simplex Table

Pivoting Column ↓

BV	X_1	X_2	S_1	S_2	S_3	Z_{max}	RHS	Ratio=RHS/PC
S_1	-1	2	1	0	0	0	8	4
S_2	1	-2	0	1	0	0	3	-3/2
S_3	1	2	0	0	1	0	12	6
z_{max}	-1	-2	0	0	0	1	0	

LV ←

Pivoting column [P.C] = Maximum $-^{ve}$ value in Z_{max} row = X_2 Column

Pivoting row [L.V] = Minimum $+^{ve}$ Ratio.=S_1 Row

Pivoting element[P.E]= Intersecting of row and column = 2

N.P.R = $X_2 = \frac{1}{PE}$ x [old row]

$= \frac{1}{2}$ x [-1, 2, 1, 0, 0, 0, 8]

$= \left(\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4\right)$

Key Element =Number marked by Pivoting column in the particular row

Key element for S_2 is -2

New Row[NR] S_2 = -Key x NPR +Old row

$= -(-2)\ [\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4] + [1, -2, 0, 1, 0, 0, 3]$

$= 2\ [\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4] + [1, -2, 0, 1, 0, 0, 3]$

$= [\frac{-2}{2}, 2, \frac{2}{2}, 0, 0, 0, 8] + [1, -2, 0, 1, 0, 0, 3]$

$= [-1, 2, 1, 0, 0, 0, 8] + [1, -2, 0, 1, 0, 0, 3]$

$= [-1+1, 2-2, 1+0, 0+1, 0+0, 0+0, 8+3]$

New Row[NR] S_2 = [0, 0, 1, 1, 0, 0, 11]

New row =NR=S_3

Key element for S_3 is 2

New Row[NR]=S_3= - Key x NPR+ Old row

$$= -2\left(\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4\right) + (1, 2, 0, 0, 1, 0, 12)$$

$$= (1, -2, -1, 0, 0, 0, -8) + (1, 2, 0, 0, 1, 0, 12)$$

$$= (1+1,\ -2+2,\ -1+0,\ 0+0,\ 0+1,\ 0+0,\ -8+12)$$

$$= [2,\ 0,\ -1,\ 0,\ 1,\ 0,\ 4]$$

New row = NR = Z_{max}

Key element for Z_{max} is -2

Z_{max} = -Key x NPR+ old row

$$= -(-2)\left(\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4\right) + (-1, -2, 0, 0, 0, 1, 0)$$

$$= 2\left(\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4\right) + (-1, -2, 0, 0, 0, 1, 0)$$

$$= (-1, 2, 1, 0, 0, 0, 8) + (-1, -2, 0, 0, 0, 1, 0)$$

$$= (-1-1,\ 2-2,\ 1+0,\ 0+0,\ 0+0,\ 0+1,\ 8+0)$$

$$= (-2,\ 0,\ 1,\ 0,\ 0,\ 1,\ 8)$$

PC ↓ (X_1)

BV	X_1	X_2	S_1	S_2	S_3	Z_{max}	RHS	Ratio=RHS/PC
X_2	$\frac{-1}{2}$	1	$\frac{1}{2}$	0	0	0	4	$\frac{4}{-1/2} = \frac{4*2}{-1} = \frac{8}{-1} = -8$
S_2	0	0	1	1	0	0	11	
LV ← S_3	2	0	-1	0	1	0	4	2
z_{max}	-2	0	1	0	0	1	8	

Pivoting element [P.E] =2

N.P.R = X_1= $\frac{1}{PE}$*[Old Row]

$X_1 = \frac{1}{2}$*[2, 0, −1, 0, 1, 0, 4]

X_1= [1, 0, $\frac{-1}{2}$, 0, $\frac{1}{2}$, 0, 2]

Key element of $x_2 = \frac{-1}{2}$

New Row x_2= -Key x NPR +Old Row

$$=-(\frac{-1}{2})\left[1, 0, \frac{-1}{2}, 0, \frac{1}{2}, 0, 2\right]+\left[\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4\right]$$

$$= \frac{1}{2}\left[1, 0, \frac{-1}{2}, 0, \frac{1}{2}, 0, 2\right]+\left[\frac{-1}{2}, 1, \frac{1}{2}, 0, 0, 0, 4\right]$$

$$= \left[\frac{1}{2}, 0, \frac{-1}{4}\ 0, \frac{1}{4}, 0,1\right]+\left[\frac{-1}{2}, 1, \frac{1}{2}, 0, 0,0, 4\right]$$

$$= \left[\frac{1}{2}-\frac{1}{2}, 0+1, \frac{-1}{4}+\frac{1}{2}, 0+0, \frac{1}{4}+0, 0+0, 1+4\right]$$

$$= \left[0, 1, \frac{1}{4}, 0, \frac{1}{4}, 0, 5\right]$$

New Row S_2 = Key element = 0

New Row S_2 = -Key x NPR + Old Row

$$= 0\left[1, 0, \frac{-1}{2}, 0, \frac{1}{2}, 0, 2\right]+\left[0, 0, 1, 1, 0, 0, 11\right]$$

$$= \left[0,0,0,0,0,0,0\right]+\left[0, 0, 1, 1, 0, 0, 11\right]$$

$$= \left[0,0,1,1,0,0,11\right]$$

New Row = Z_{max} key element = -2

New Row =Z_{max} = -Key x NPR + Old

$$= -(-2)\left[1, 0, \frac{-1}{2}, 0, \frac{1}{2}, 0, 2\right]+\left[-2, 0, 1, 0, 0, 1, 8\right]$$

$$= 2\left[1, 0, \frac{-1}{2}, 0, \frac{1}{2}, 0, 2\right]+\left[-2, 0, 1, 0, 0, 1, 8\right]$$

$$= \left[2, 0, -1, 0, 1, 0, 4\right]+\left[-2, 0, 1, 0, 0, 1, 8\right]$$

$$= \left[2-2, 0+0, -1+1, 0+0, 1+0, 0+1, 4+8\right]$$

$$= \left[0\quad 0\quad 0\quad 0\quad 1\quad 1\quad 12\right]$$

BV	X_1	X_2	S_1	S_2	S_3	Z_{max}	RHS	Ratio=RHS/PC
X_2	0	1	$\frac{1}{4}$	0	$\frac{1}{4}$	0	5	
S_2	0	0	1	1	0	0	11	
X_1	1	0	$\frac{-1}{2}$	0	$\frac{1}{2}$	0	2	
z_{max}	0	0	0	0	1	1	12	

$1X_2$= 5, $1X_1$=2 , $1Z_{max}$=12

$\boxed{X_1=2}$ $\boxed{X_2=5}$ $\boxed{Z_{max}=12}$

Simplex Method

Q3) Solve : $Z_{max} = 2x_1 + 4x_2 + 3x_3$
S.T $x_1 + x_2 + x_3 <= 18$
$4x_1 + 2x_2 + x_3 <= 35$

Solution: $x_1 + x_2 + x_3 + s_1 = 18$
$4x_1 + 2x_2 + x_3 + s_2 = 35$
$Z_{max} - 2x_1 - 4x_2 - 3x_3 = 0$

Basic variable Unique,Unit ,+^{ve} co-efficient S_1 , S_2, Z_{max}
I.B.F.S Let x_1, x_2, x_3 are zero then $S_1 = 18$, $S_2 = 35$, $Z_{max} = 0$

Pivoting Column ↓ PC

Simplex Table

BV	X_1	X_2	X_3	S_1	S_2	Z_{max}	RHS	Ratio=RHS/PC
S_1	1	1	1	1	0	0	18	$\frac{18}{1}=18$
S_2	4	2	1	0	1	0	35	$\frac{35}{2}=17.5$
Z_{max}	-2	-4	-3	0	0	1	0	

LV ←

Pivoting Element[P.E]= Intersecting of row and column = 2

New Pivoting Row [N.P.R] = $\frac{1}{PE}$ x [Old row]

$$= \frac{1}{2} \times \left(4, 2, 1, 0, 1, 0, 35\right)$$

$$= \left(\frac{4}{2}, \frac{2}{2}, \frac{1}{2}, \frac{0}{2}, \frac{1}{2}, \frac{0}{2}, \frac{35}{2}\right)$$

$$NPR = x_2 = \left(2,\ 1, \frac{1}{2},\ 0, \frac{1}{2}, 0, \frac{35}{2}\right)$$

Key Element of S_1 is 1

New Row[NR] S_1 = -Key x NPR +Old row

$$= -1\ [2, 1, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{35}{2}] + [1,1,1,1,0,0,18]$$

$$= [-2, -1, \frac{-1}{2}, 0, \frac{-1}{2}, 0, \frac{-35}{2}] + [1,1,1,1,0,0,18]$$

$$= [-2+1, -1+1, \frac{-1}{2}+1, 0+1, \frac{-1}{2}+0, 0+0, \frac{-35}{2}+18]$$

$$= [-1 \quad 0 \quad \frac{1}{2} \quad 1 \quad \frac{-1}{2} \quad 0 \quad \frac{1}{2}]$$

<u>Simplex Method</u>

Key element of Z_{max} is -4

New Row[NR]= Z_{max} = - Key x NPR+ Old row

$$= -(-4) \times \left(2, 1, \tfrac{1}{2}, 0, \tfrac{1}{2}, 0, \tfrac{35}{2}\right) + \left(-2, -4, -3, 0, 0, 1, 0\right)$$

$$= 4\left(2, 1, \tfrac{1}{2}, 0, \tfrac{1}{2}, 0, \tfrac{35}{2}\right) + \left(-2, -4, -3, 0, 0, 1, 0\right)$$

$$= \left(8, 4, 2, 0, 2, 0, 70\right) + \left(-2, -4, -3, 0, 0, 1, 0\right)$$

$$= \left(8-2,\ 4-4,\ 2-3,\ 0+0,\ 2+0,\ 0+1,\ 70+0\right)$$

$$= [\,6 \quad 0 \quad -1 \quad 0 \quad 2 \quad 1 \quad 70\,]$$

Simplex Table

P.E ↓ (X_3 column)

BV	X_1	X_2	X_3	S_1	S_2	Z_{max}	RHS	Ratio=RHS/PC
← S_1	-1	0	$\frac{1}{2}$	1	$\frac{-1}{2}$	0	$\frac{1}{2}$	$\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} * \frac{2}{1} = 1$
X_2	2	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{35}{2}$	$\frac{\frac{35}{2}}{\frac{1}{2}} = \frac{35}{2} * \frac{2}{1} = 35$
z_{max}	6	0	-1	0	2	1	70	-71

Pivoting Element = [P.E] = Intersect of Row column = $\frac{1}{2}$

New Pivoting Row [NPR] = X_3

$X_3 = \frac{1}{PE}$ x [old row]

$$= \frac{1}{\frac{1}{2}}\left(-1, 0, \tfrac{1}{2}, 1, \tfrac{-1}{2}, 0, \tfrac{1}{2}\right)$$

$$= 2\left(-1, 0, \tfrac{1}{2}, 1, \tfrac{-1}{2}, 0, \tfrac{1}{2}\right)$$

$$= \left(-2, 0, 1, 2, -1, 0, 1\right)$$

Key Element of $X_2 = \frac{1}{2}$

New Row [NR] $=X_1=$ -Key x NPR +Old Row

$$= -\frac{1}{2}\left[-2, 0, 1, 2, -1, 0, 1\right] + \left[2, 1, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{35}{2}\right]$$

$$= \left[1, 0, \frac{-1}{2}, -1, \frac{1}{2}, 0, \frac{-1}{2}\right] + \left[2, 1, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{35}{2}\right]$$

$$= \left[1+2,\ 0+1, \frac{-1}{2}+\frac{1}{2},\ -1+0,\ \frac{1}{2}+\frac{1}{2}, 0+0,\ \frac{-1}{2}+\frac{35}{2}\right]$$

$$X_2 = \left[\ 3 \quad 1 \quad 0 \quad -1 \quad 1 \quad 0 \quad 17\right]$$

Key Element 0f $Z_{max} = -1$

NR = $Z_{max=}$ -Key x NPR + Old Row

$$= -(-1)\left[-2, 0, 1, 2, -1, 0, 1\right] + \left[6, 0, -1, 0, 2, 1, 70\right]$$

$$= \ 1\left[-2, 0, 1, 2, -1, 0, 1\right] + \left[6, 0, -1, 0, 2, 1, 70\right]$$

$$= \left[-2, 0, 1, 2, -1, 0, 1\right] + \left[6, 0, -1, 0, 2, 1, 70\right]$$

$$= \left[-2+6, 0+0, 1-1, 2+0, -1+2,\ 0+1,\ 1+70\right]$$

$$= \left[4 \quad 0 \quad 0 \quad 2 \quad 1 \quad 1 \quad 71\right]$$

BV	X_1	X_2	X_3	S_1	S_2	Z_{max}	RHS	Ratio=RHS/PC
X_3	-2	0	1	2	-1	0	1	
X_2	3	1	0	-1	1	0	17	
Z_{max}	4	0	0	2	1	1	71	

$X_2 = 17$ $\quad X_3=1$ $\quad X_1=0$ $\quad Z_{max}=71$

Transportation Problem

Production Unit	S_1	S_2	S_3	S_4	Available
P_1	6	4	1	5	14
P_2	8	9	2	7	16
P_3	4	3	6	2	5
Demand	6	10	15	4	

Solution:

Allotment for the above problem using Matrix minima method

Production Unit	S_1	S_2	S_3	S_4	Available
P_1	6	4	14 1	5	14
P_2	6 8	9 9	1 2	7	16
P_3	4	1 3	6	4 2	5
Demand	6	10	15	4	

T.T.C:- 14 * 1 + 6 * 8 + 9 * 9 + 15 * 2 + 1 * 3 + 4 * 2

14 + 48 + 81 + 30 + 3 + 8

= 156

m= Number of rows=3

n=Number of columns=4

m + n – 1

3+4-1 = 6

U .V Method

Ui →

Vj ↓

	7	8	1	7
0			1	
1	8	9	2	
-5		3		2

Cij = (ui + vj)

$C_{ij} = u_i + v_j$
$1 = u_i + 0$
$u_i = 1$

$C_{ij} = u_i + v_j$
$2 = 1 + v_j$
$v_j = 1$

$C_{ij} = u_i + v_j$
$8 = u_i + 1$
$u_i = 7$

$C_{ij} = u_i + v_j$
$9 = u_i + 1$
$u_i = 8$

$C_{ij} = u_i + v_j$
$3 = 8 + v_j$
$v_j = -5$

$C_{ij} = u_i + v_j$
$2 = u_i - 5$
$u_i = 7$

Cij – (ui + vj) > 0

	7	8	1	7
0	6	4		5
1				7
-5	4		6	

6 – (7+0)

6–7 = -1 > 0
False
Negative

4 – (8 + 0)

4 – 8 = -4 > 0
False

Negative

5 – (7 + 0)

5 – 7 = -2 > 0
False
Negative

7 – (1 + 7)

7 – 8 = -1 > 0
False
Negative

4 – (-5 + 7)

4 – (2) = 2 > 0

6 – (-5 + 1)

6 –(-4) = 6+4 =10 > 0

Modi Method

Choose Maximum Negative cell (Here 1st row second column)

	+ ϴ 4	14 - ϴ 1	
6 8	9- ϴ 9	1+ ϴ 2	
	1 3		4 2

Min $\{14 - \theta, 9 - \theta\}$

9 – ϴ = 0

ϴ = 9

6	9 4	5 1	5
6 8	9	10 2	7
4	1 3	6	4 2

T.T.C = 9 * 4 + 5 * 1 + 6 * 8 + 10 * 2 + 1 * 3 + 4 * 2

= 36 + 5 + 48 + 20 + 3 + 8

= 120

UV method

6	9 4	5 1	5	14
6 8	9	10 2	7	16
4	1 3	6	4 2	5
6	10	15	4	

M + N – 1

3 + 4 – 1

= 6

$$C_{ij} = u_i + v_j$$

Ui →

Vj ↓

	7	4	1	3
0		4	1	
1	8		2	
-1		3		2

$C_{ij} = u_i + v_j$
$4 = u_i + 0$
$u_i = 4$

$C_{ij} = u_i + v_j$
$1 = u_i + 0$
$v_j = 1$

$C_{ij} = u_i + v_j$
$2 = 1 + v_j$
$v_j = 1$

$C_{ij} = u_i + v_j$
$8 = u_i + 1$
$u_i = 7$

$C_{ij} = u_i + v_j$
$3 = 4 + v_j$
$v_j = -1$

$C_{ij} = u_i + v_j$
$2 = u_i + (-1)$
$2 = u_i - 1$
$u_i = 3$

	7	4	1	3
0	6			5
1		9		7
-1	4		6	

$C_{ij} - (u_i + v_j) > 0$

$6 - (0 + 7)$
$6 - 7 = -1 > 0$
False, Negative

$5 - (3 + 0)$

$5 - 3 = 2 > 0$

$4 - (-1 + 7)$
$4 - 6 = -2 > 0$
False, Negative

$9 - (4 + 1)$
$9 - 5 = 4 > 0$

$7 - (1 + 3)$
$7 - 4 = 3 > 0$

$6 - (-\cancel{1} + \cancel{1})$
$= 6 > 0$

Modi Method

	9+θ 4	5- θ 1	
6 - θ 8		10+ θ 2	
+ θ 4	1 - θ 3		4 2

Min $\{6-\theta, 5-\theta, 1-\theta\}$

$$\mathbf{1-\theta=0}$$
$$\mathbf{\theta=1}$$

6	10 4	4 1	5	14
5 8	9	11 2	7	16
1 4	3	6	4 2	5
6	10	15	4	

T.T.C = 10 * 4 + 4 * 1 + 5 * 8 + 11 * 2 + 1 * 4 + 4 * 2

= 40 + 4 + 40 + 22 + 4 + 8

= 118

U V method

6	10 4	4 1	5	14
5 8	9	11 2	7	16
1 4	3	6	4 2	5
6	10	15	4	

M + N – 1

3 + 4 – 1

= 6

Cij = (ui + vj)

Ui →

Vj ↓

	7	4	1	5
0		4	1	
1	8		2	
-3	4			2

$C_{ij} = u_i + v_j$
$4 = u_i + 0$
$u_i = 4$

$C_{ij} = u_i + v_j$
$1 = u_i + 0$
$v_j = 1$

$C_{ij} = u_i + v_j$
$2 = 1 + v_j$
$v_j = 1$

$C_{ij} = u_i + v_j$
$8 = u_i + 1$
$u_i = 7$

$C_{ij} = u_i + v_j$
$4 = 7 + v_j$
$v_j = -3$

$C_{ij} = u_i + v_j$
$2 = u_i + (-3)$
$2 = u_i - 3$
$u_i = 5$

	7	4	1	5
0	6			5
1		9		7
-3		3	6	

6 – (0 + 7)
6 – 7 = - 1 > 0
False Negative

5 – (5 + 0)

5 – 5 = 0 > 0

9 – (1 + 4)
9 – 5 = 4 > 0
3 – (-3 + 4)
3 – (1) = 2 > 0

7 – (1 + 5)
7 – 6 = 1 > 0
6 – (-3 + 1)
6 – (-2) = 8 > 0

<u>Modi Method</u>

$+\theta$ 6	(10) 4	($4-\theta$) 1	
($5-\theta$) 8		($11+\theta$) 2	
(1) 4			(4) 2

$$\text{Min}\left\{\ 4-\theta\ ,\ 5-\theta\ \right\}$$

$$\mathbf{4-\theta=0}$$

$$\mathbf{\theta=4}$$

(4) 6	(10) 4	1	5	14
(1) 8	9	(15) 2	7	16
(1) 4	3	6	(4) 2	5
6	10	15	4	

T.T.C = 4 * 6 + 10 * 4 + 1 * 8 + 15* 2 + 1 * 4 + 4 * 2

= 24 + 40 + 8 + 30 + 4 + 8

= 114

UV method

Ui → / Vj ↓	6	4	0	4
0	6	4		
2	8		2	
-2	4			2

M + N – 1

3 + 4 – 1

= 6

$C_{ij} = u_i + v_j$
$6 = u_i + 0$
$u_i = 6$

$C_{ij} = u_i + v_j$
$4 = u_i + 0$
$v_j = 4$

$C_{ij} = u_i + v_j$
$8 = 6 + v_j$
$v_j = 2$

$C_{ij} = u_i + v_j$
$2 = u_i + 2$
$u_i = 0$

$C_{ij} = u_i + v_j$
$4 = 6 + v_j$
$v_j = -2$

$C_{ij} = u_i + v_j$
$2 = u_i + (-2)$
$u_i = 4$

	6	4	0	4
0			1	5
2		9		7
-2		3	6	

Cij – (ui + vj) > 0

1 – (0 + 0) **5 – (0 + 4)** **9 – (2 + 4)** **7 – (2 + 4)**

1 – 0 = 1 > 0 **5 – 4 = 1 > 0** **9 – 6 = 3 > 0** **7 – 6 = 1 > 0**

3 – (-2 + 4) **6 – (-2 + 0)**

3 – (2) = 1 > 0 **6 + 2 = 8 > 0**

All values are possitive, so answer obtain is optimal.

Answer 114

Transportation Problem – VAM Method

Voggel's Approximation Method[V.A.M]

Steps:

1) Calculating Penalty

It is an absolute difference between 2 Least cells in each Row and Column

2) Allot in the column / Row which is having highest penalty, in that allot the cell which is having least value

   (If the tie between penalty, then choose the cell where we can assign maximum units)

3) Re caluclate penalty before going to next allotment
4) Repeat step 1 to 3 untill supply and demand fullfill.

Production unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	4	1	5	**14**	
C. H. Nagar	8	9	2	7	**16**	
Mandya	4	3	6	2	**5**	
Demand	**6**	**10**	**15**	**4**		

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	4	1	5	**14**	3
C. H. Nagar	8	9	2	7	**16**	**5**
Mandya	4	3	6	2	**5**	1
Demand	**6**	**10**	**15**	**4**		
Penalty	2	1	1	3		

Transportation Problem – VAM Method

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	4	1	5	**14**	
C. H. Nagar	8	9	15 2	7	**~~16~~ 1**	**5**
Mandya	4	3	6	2	**5**	
Demand	**6**	**10**	**~~15~~ 0**	**4**		
Penalty						

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	4	X 1	5	**14**	1
C. H. Nagar	8	9	15 2	7	**~~16~~ 1**	1
Mandya	4	3	X 6	2	**5**	1
Demand	**6**	**10**	**~~15~~ 0**	**4**		
Penalty	2	1	X	3		

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	4	X 1	5	**14**	
C. H. Nagar	8	9	15 2	7	**~~16~~ 1**	
Mandya	4	3	X 6	4 2	**~~5~~ 1**	
Demand	**6**	**10**	**~~15~~ 0**	**~~4~~ 0**		
Penalty				**3**		

Transportation Problem – VAM Method

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	4	X 1	X 5	**14**	**2**
C. H. Nagar	8	9	15 2	X 7	**~~16~~ 1**	1
Mandya	4	3	X 6	4 2	**~~5~~ 1**	1
Demand	**6**	**10**	**~~15~~ 0**	**~~4~~ 0**		
Penalty	**2**	1	X	X		

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	10 4	X 1	X 5	**~~14~~ 4**	**2**
C. H. Nagar	8	9	15 2	X 7	**~~16~~ 1**	
Mandya	4	3	X 6	4 2	**~~5~~ 1**	
Demand	**6**	**~~10~~ 0**	**~~15~~ 0**	**~~4~~ 0**		
Penalty						

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	10 4	X 1	X 5	**~~14~~ 4**	6
C. H. Nagar	8	X 9	15 2	X 7	**~~16~~ 1**	**8**
Mandya	4	X 3	X 6	4 2	**~~5~~ 1**	4
Demand	**6**	**~~10~~ 0**	**~~15~~ 0**	**~~4~~ 0**		
Penalty	2	X	X	X		

Transportation Problem – VAM Method

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	10 4	X 1	X 5	~~14~~ 4	
C. H. Nagar	1 8	X 9	15 2	X 7	~~16~~ ~~1~~ 0	8
Mandya	4	X 3	X 6	4 2	~~5~~ ~~1~~ 0	
Demand	~~6~~ 5	~~10~~ 0	~~15~~ 0	~~4~~ 0		
Penalty						

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	6	10 4	X 1	X 5	~~14~~ 4	6
C. H. Nagar	1 8	X 9	15 2	X 7	~~16~~ ~~1~~ 0	X
Mandya	4	X 3	X 6	4 2	~~5~~ 1	4
Demand	~~6~~ 5	~~10~~ 0	~~15~~ 0	~~4~~ 0		
Penalty	2	X	X	X		

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	4 6	10 4	X 1	X 5	~~14~~ ~~4~~ 0	6
C. H. Nagar	1 8	X 9	15 2	X 7	~~16~~ ~~1~~ 0	
Mandya	4	X 3	X 6	4 2	~~5~~ 1	
Demand	~~6~~ ~~5~~ 1	~~10~~ 0	~~15~~ 0	~~4~~ 0		
Penalty						

Transportation Problem – VAM Method

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	4 6	10 4	X 1	X 5	**~~14~~ ~~4~~ 0**	X
C. H. Nagar	1 8	X 9	15 2	X 7	**~~16~~ ~~1~~ 0**	X
Mandya	4	X 3	X 6	4 2	**~~5~~ 1**	4
Demand	**~~6~~ ~~5~~ 1**	**~~10~~ 0**	**~~15~~ 0**	**~~4~~ 0**		
Penalty	**4**	X	X	X		

Production Unit	Maddur	K.R Nagar	N. Gudu	P. Pura	Available	Penalty
Mysuru	4 6	10 4	X 1	X 5	**~~14~~ ~~4~~ 0**	
C. H. Nagar	1 8	X 9	15 2	X 7	**~~16~~ ~~1~~ 0**	
Mandya	1 4	X 3	X 6	4 2	**~~5~~ ~~1~~ 0**	**4**
Demand	**~~6~~ ~~5~~ ~~1~~ 0**	**~~10~~ 0**	**~~15~~ 0**	**~~4~~ 0**		
Penalty	**4**					

Total transportation cost

4*6 + 10*4 + 1*8 + 15*2+ 1*4+ 4*2= 24 + 40 + 8 + 30 + 4 +8 = 114