Chapter 6: Interference of light by division of wave front

Syllabus: Huygen's theory - Concept of wave front, Interference pattern produced on the surface of water, Coherence, Conditions for Interference, Interference of light waves by division of wave-front, Fresnel Biprism - derivation of expression for fringe width and determination of wavelength, Interference with white light, Numerical Problems. (04 hours)

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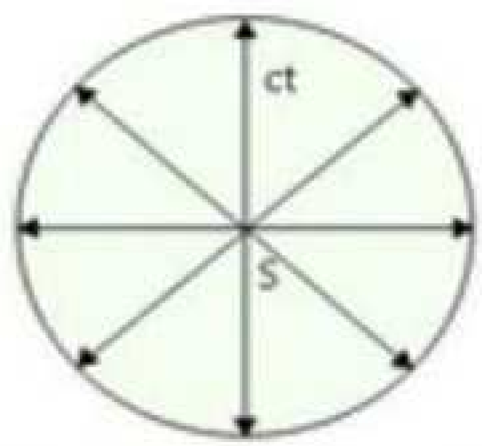
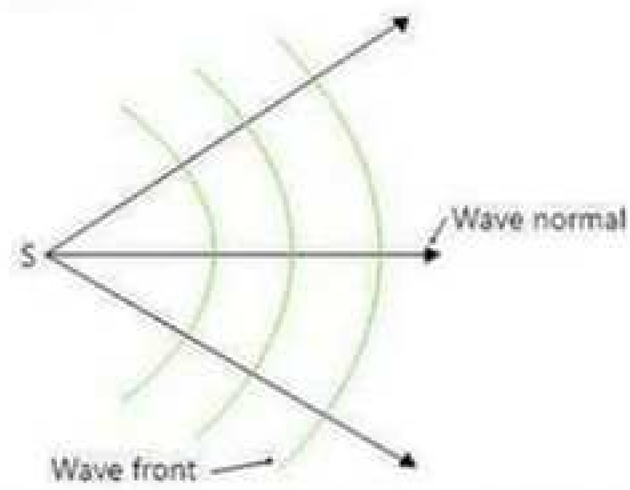
Concept of wave front:

A **wave front** is defined as **an imaginary surface** over which the phase of the wave is constant.

Consider a point source of light 'S' in 'air.' The light waves emitted by this source travel in all directions. If 'c' is the velocity of light in air, each wave will travel a distance of **'c.t'** in time 't' and reach the surface of a sphere with radius **'c.t'** (figure), with the source 'S' as its focal point. This type of surface is known as a **spherical wave surface**.





A **wave front** is a locus of all the points of medium to which waves arrive simultaneously, so that all the points are in the same phase.

Types of wave fronts – There are **three types** of wave fronts:

i) A **spherical wave front** is produces by light from a bulb nearby.
ii) A **plane wave front** emerges from distant source of light (sunlight).
iii) A linear source (such as a slit) produces a **cylindrical wave front**. For example, light is emitted by a fluorescent tube.

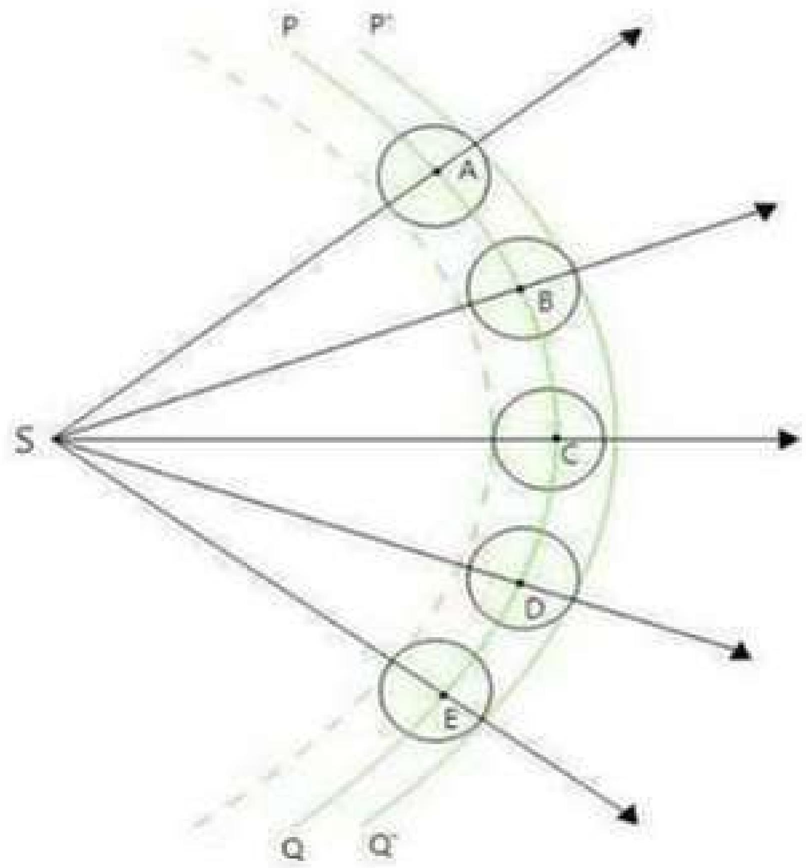
A **wave normal** is a perpendicular drawn to the surface of a wave front in the direction of light propagation. A wave front, in other words, transports light energy perpendicular to the surface.

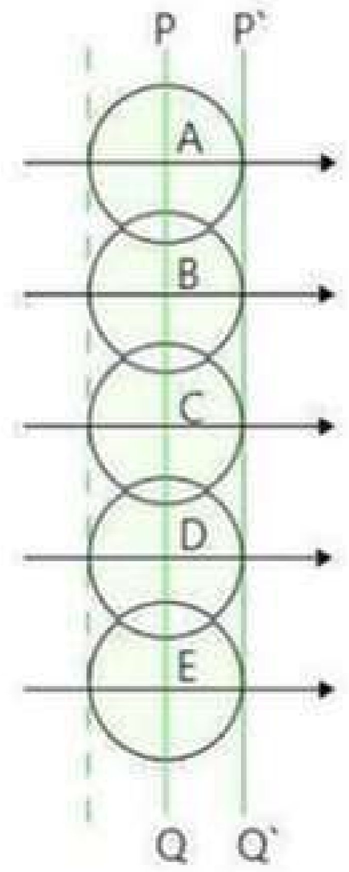
Huygen's Construction of a spherical wave front – Huygen's Principle:

Let PQ be a cross-section of a spherical wave front due to a point source (S), at any instant (Figure). This is called as **primary wave front**. Now consider points A, B, C, D, E on PQ. They act as secondary sources and send out **secondary wavelets**.

If 'c' is the speed of light in the isotropic medium, in time t, each wave will describe a distance 'ct'. With A, B, C, D, E as centres of circles, each radius **'c.t'** will be traced. Each circle will represent a **secondary wave front**.

The common tangential surface (envelope) P'Q' drawn to these secondary wave fronts represents the new position of the wave front after time 't'.





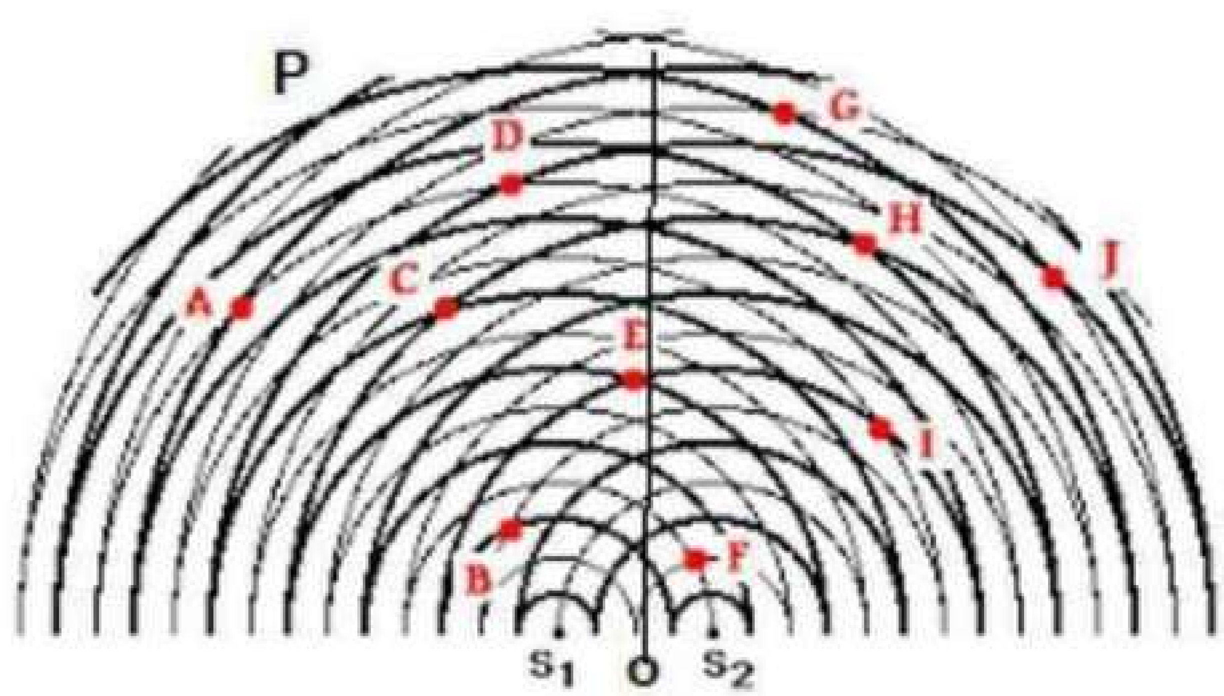
Interference pattern produced on the surface of water:

Interference is a phenomenon in which two waves combine by adding their displacement together at every single point in space and time, to form a resultant wave of greater or lower amplitude.

We consider surface waves **emanating** (originating) two point sources in a water tank. We may consider two sharp needles vibrating up and down at points S_1 *and* S_2 (figure). We assume water waves to produce displacements which are transverse to the direction of propagation.

If there is only one needle (say at S_1) vibrating with a frequency ν. Then the wavelength would be $\lambda = \frac{velocity}{frequency} = \frac{v}{\nu}$ and the crests and troughs would have moved outwards. The same might happen for the vibrating needle at S_2.

If both the needles are vibrating, then the waves emanating from S_1 will interfere with the waves emanating from S_2. It is assumed that both the needles vibrate with the same phase. At a certain instant, the waves emanating from S_1 produces a crest or trough at a distance (say 'd') from S_1, then, the waves emanating from S_2 would also produce a crest or trough at the same distance 'd' from S_2.



This is clearly shown in figure. The darker lines represent the crests produced by the waves, at a particular instant, originating from S_1 *and* S_2. Similarly, the lighter lines represent the troughs produced by these waves, at the same instant. It is obvious that all points on the bisector **OE** are in phase.

Then, at an arbitrary point **E** on the bisector, we may write the **resultant disturbance** as,

$$y = y_1 + y_2 = 2a \cos\omega t \quad \text{......................... (1)}$$

where, $y_1 = y_2 = a \cos\omega t$

Both y_1 ***and*** y_2 represent the displacements at the point E due to S_1 *and* S_2 respectively. Since there is no phase difference, **Constructive interference** takes place at E and the intensity at E will be maximum.

Next, let us consider a point at B, such that, the waves from S_1 *and* S_2 reach the point B out of phase (**phase difference of π & a time difference of T/2**). Then, **the path difference,**

$$S_2B - S_1B = \frac{\lambda}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Consequently, if the displacement at B due to S_1 is given by, $y_1 = a\, cos\omega t$.

Then, the displacement at B due to S_2 must be $y_1 = a\, cos(\omega t - \pi) = -\, a\, cos\, \omega t$

Then, the resultant disturbance, $y = y_1 + y_2 = 0$ .... (3)

The point B corresponds to **destructive interference** and is known as a **node**. At nodes, the intensity is minimum (nearly zero).

In general, at any point P is such that,

$$S_2P - S_1P = n\,\lambda \quad \textbf{(maxima)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where, **n = 0, 1, 2, 3, ….**

The disturbances reaching the point P from S_1 *and* S_2 will be **<u>in phase</u>**. This produces **constructive interference** and the **intensity will be maximum.**

On the other hand, if the point P is such that,

$$S_2P - S_1P = (n + \tfrac{1}{2})\,\lambda \quad \textbf{(minima)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where, **n = 0, 1, 2, 3, ….**

The disturbances reaching the point P from S_1 *and* S_2 will be **<u>out of phase</u>**. This produces **destructive interference** and the **intensity will be minimum.**

<u>Interference of Light:</u>

Interference is a phenomenon in which two waves superpose to form a resultant wave of greater or lower amplitude. Interference effects can be observed with all types of waves, for example, light, radio, sound, surface water waves, gravity waves.

The **principle of superposition of waves** states that when two or more propagating waves of the same type are incident on the same point, the resultant amplitude at that point is equal to the vector sum of the amplitudes of the individual waves.

When two waves which are in phase (i.e., no phase difference) at the same point, then the amplitude is the sum of the individual amplitudes. This is called **constructive interference**. Since the intensity of light get added up, **bright fringes** are observed.

When two waves which are out of phase (i.e., phase difference is π) at the same point, then the amplitude is the difference of the individual amplitudes. This is called **destructive interference**. Since the intensity decreases, **dark fringes** are observed.

<u>Coherent Sources:</u> Two sources are said to be coherent when the waves emitted from them have the **same frequency and constant phase difference.**

Conditions for Interference:

For continuous interference of light to occur, the following conditions must be met:

1. **Coherent sources** of light are needed.
2. Amplitudes and intensities must be nearly equal to produce sufficient contrast between maxima and minima.
3. The source must be small enough that it can be considered a point source of light.
4. The interfering sources must be nearby to produce wide fringes.
5. The source and screen must be far enough to produce wide fringes.
6. The sources must be monochromatic.

Examples for Coherent Sources:

i) **Laser light** is an example of coherent source of light.

ii) **Sound waves** are another example of coherent sources.

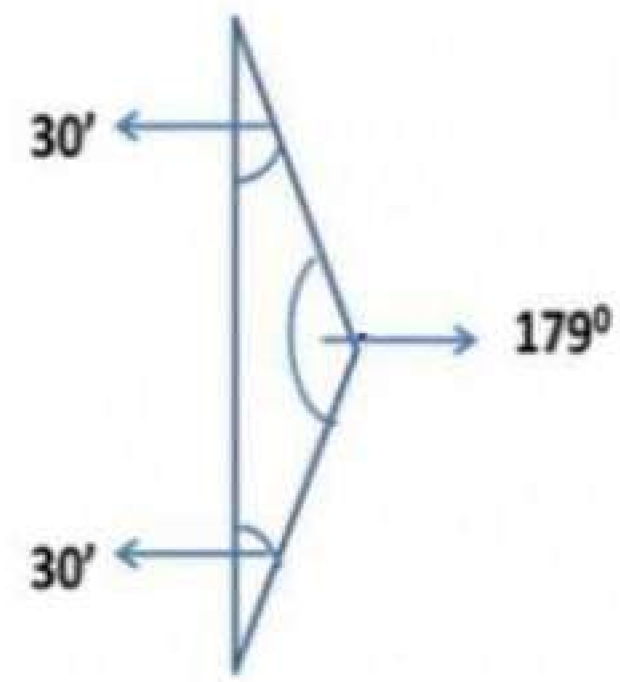
Note: The phenomenon of interference may be grouped into two categories, namely, **Division of Wave front** and **Division of Amplitude**.

Division of Wave front: Under this category, the coherent sources are obtained by dividing the wave front, originating from a common source, by employing 2 pin holes, mirrors, biprism etc. This type of interference requires a **point source** or a **narrow slit** source.

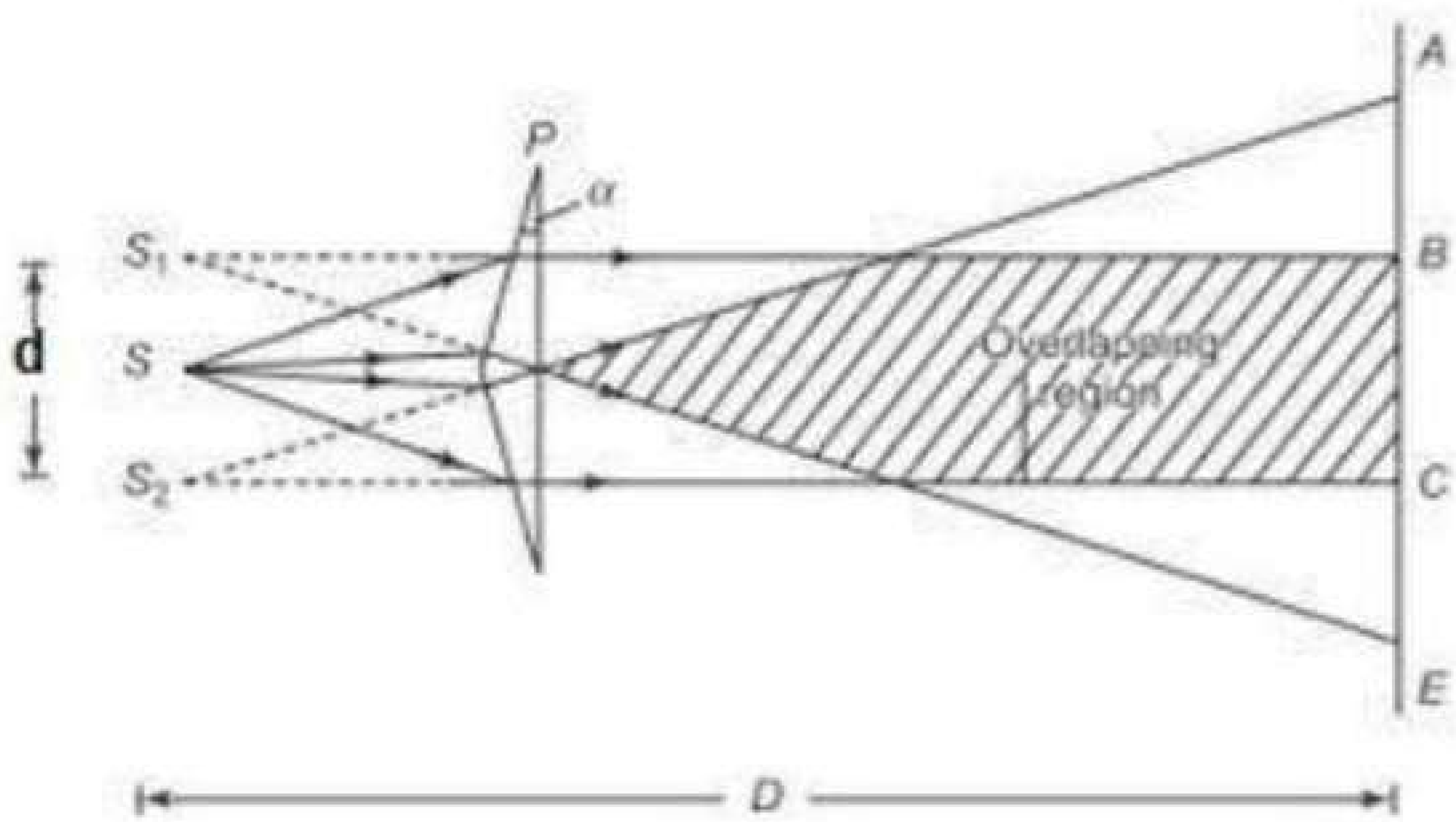
The experiments employed to obtain interference by division of wavefront are the Young's double slit experiment, Fresnel biprism, Lloyd's mirror etc.

Fresnel Biprism:

Fresnel Biprism consists of two acute angled prisms with their bases in contact. In actual practice, the bi-prism is constructed as a single prisms of **obtuse angle of about 179° and the remaining two acute angles are 30' each**. When a monochromatic light source is kept in front of bi-prism two coherent virtual sources are produced.



Here, the interference is observed by the **division of wave front**.



From a Monochromatic source, light through a narrow slit S falls on biprism, which divides it into two components. One of these components is refracted from upper portion of biprism and appears to come from S_1, while the other one is refracted through lower portion and appears to come from S_2. Thus, S_1 and S_2 act as **two virtual coherent sources** formed from the original source.

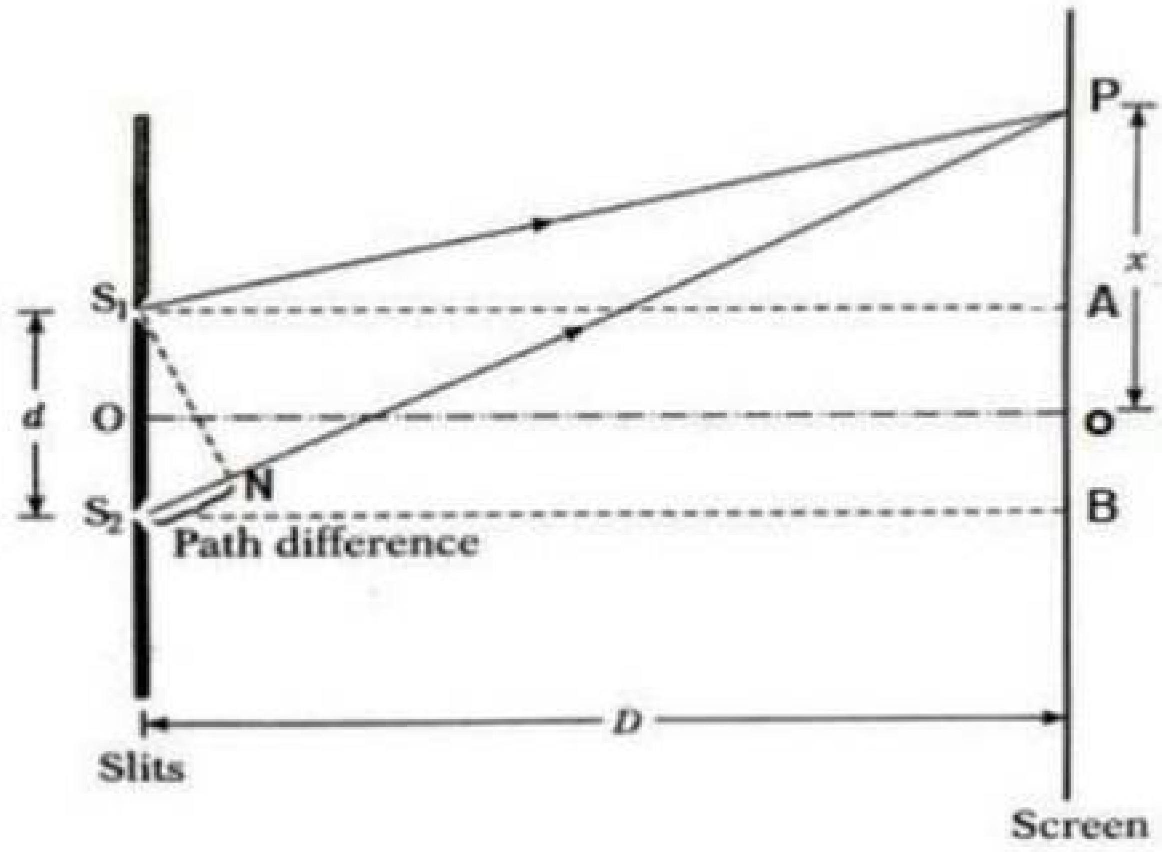
Light waves arising from S_1 and S_2 interfere in the shaded region and interference fringes are formed can be observed on the screen. (Fig.) It is observed that the bright and dark fringes are alternate and equally spaced.

Expression for fringe width (β):

Fringe width is the distance between two consecutive bright or dark fringes, i.e., fringe width is the width of one bright or dark fringe.

Fresnel's biprism experiment is similar to Young's double slit experiment.

Interference pattern is formed on a screen using two monochromatic point sources S_1 and S_2 which are in **same phase**. These monochromatic virtual sources S_1 and S_2 are separated by a distance 'd'. A screen is placed at a distance of D from the slits. *Also,* $\boldsymbol{AB = d}$ *and* $\boldsymbol{OA = OB = \frac{d}{2}}$.



Let P be a point on the screen, distant **'x'** from the centre O.

Constructive interference to get a bright fringe at P:

Let $\boldsymbol{n^{th}}$ bright fringe is obtained at P, distant $\boldsymbol{x_n}$ from the centre O. We know that, the condition for path difference to get a bright fringe is $\boldsymbol{n\lambda}$. The **path difference** between the paths, S_1P *and* S_2P *is* $\boldsymbol{S_2N}$.

$$\text{i.e., } path\ difference,\ S_2N = S_2P - S_1P = n\lambda \quad \ldots\ldots\ldots\ldots (1)$$

where, n = 0, 1, 2, ...

From figure,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x_n - \frac{d}{2}\right)^2\right]$$

$$= \left[D^2 + x_n^2 + x_n d + \frac{d^2}{4}\right] - \left[D^2 + x_n^2 - x_n d + \frac{d^2}{4}\right]$$

$$= \left[D^2 + x_n^2 + x_n d + \frac{d^2}{4} - D^2 - x_n^2 + x_n d - \frac{d^2}{4}\right]$$

$$(S_2P)^2 - (S_1P)^2 = 2x_n d$$

$$(S_2P - S_1P).(S_2P + S_1P) = 2x_n d$$

$$(S_2P - S_1P) = \frac{2x_n d}{(S_2P + S_1P)}$$

$$path\ difference = \frac{2x_n d}{(S_2P + S_1P)}$$

But, $S_2P \approx S_1P = D.$

Then, $$path\ difference = \frac{2x_n d}{2D} = \frac{x_n d}{D}$$

$$n\lambda = \frac{x_n d}{D}$$

or $$x_n = \frac{n\lambda D}{d} \quad \text{.......................... (1)}$$

Similarly, for $(n-1)^{th}$ **bright fringe**, we write, $x_{n-1} = \frac{(n-1)\lambda D}{d}$ .... (2)

From eqns. (1) and (2), we get,

$$x_n - x_{n-1} = \frac{n\,\lambda\,D}{d} - \frac{(n-1)\lambda\,D}{d}$$

$$\textbf{fringe width, } \beta = \frac{\lambda\,D}{d}$$

where **D** is the distance between the virtual sources and the screen, **'d'** is the separation between the two virtual sources.

Note: Since the fringes are equally spaced, the distance between two consecutive bright or consecutive dark fringes is the fringe width.

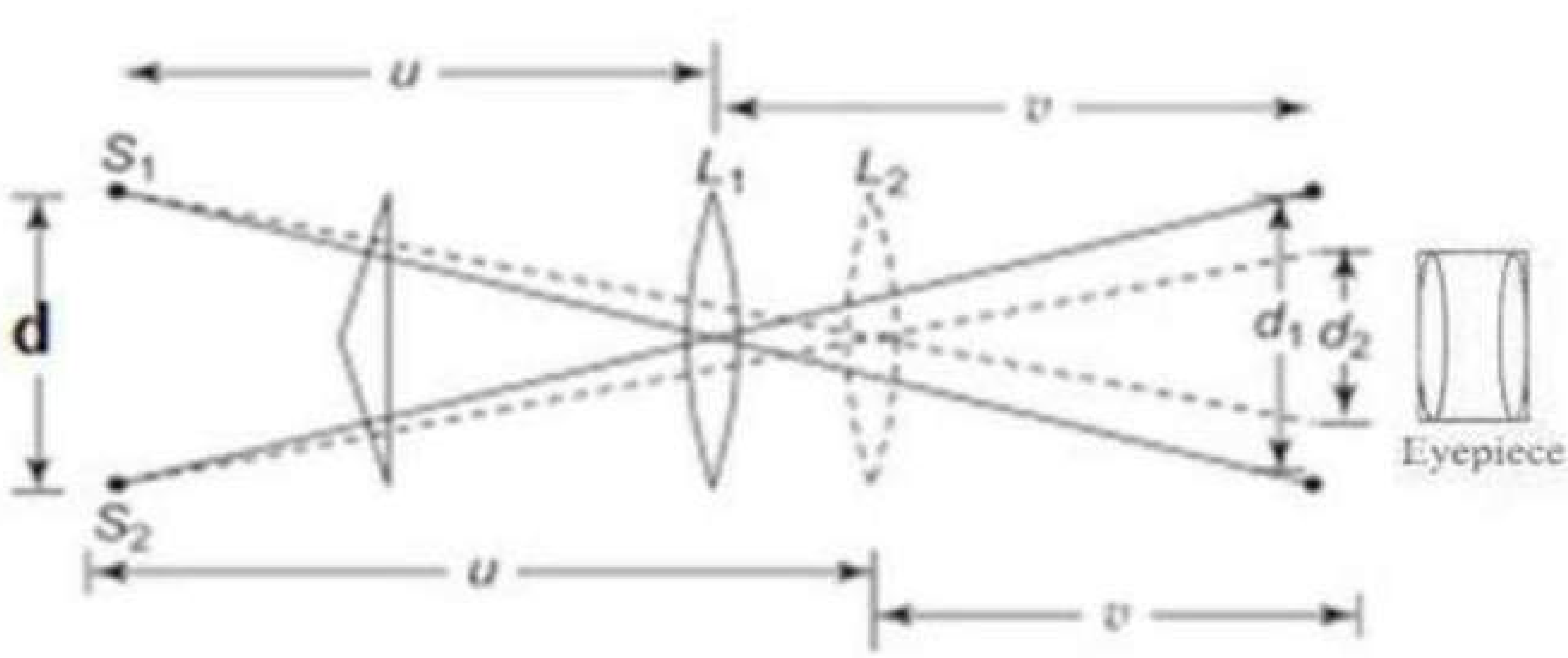
Determination of wavelength of the source (λ) of the source using Biprism:

When the light is observed on a screen from a distance D away from a biprism, the resulting interference pattern is a series of light and dark fringes. It can be shown from the geometry that the fringe width, β, is given as,

$\beta = \frac{\lambda\,D}{d}$, where **λ** is the wavelength of the light source and 'd' is the distance between the two virtual sources $S_1 and\ S_2$ of the biprism. (Figure).

This experiment is very much similar to Young's double slit experiment, but, the value '**d**' cannot be measured directly, as the sources are virtual in nature.

To find 'd': Here, in order to get real images of the virtual slits, a convex lens is introduced between the biprism and the eyepiece (screen). At two different distinct positions of the convex lens, say $L_1 and\ L_2,$ a **diminished image** and an **enlarged image** are seen clearly.



Using **conjugate foci** method, the distance between the two virtual images (**d**) can be calculated as follows:

In conjugate foci method, when the object distance (**u**) and image distance (**v**) are interchanged, **a diminished image and an enlarged real image** are seen distinctly.

Here, **D = u + v**.

When the convex lens is at L_1, a clear **magnified image** of the two virtual sources (slits) is seen. Let the object distance be u_1 and image distance be v_1. Obviously, $u_1 + v_1 = D$. Using a micro meter, the distance between the two slits (d_1) is measured.

Now, for an another position of the convex lens $L_{2,}$ with the object distance and the image distance interchanged, such that, $u_2 = v_1$ *and* $v_2 = u_{1,}$ again a clear **diminished image** of the two virtual sources (slits) is seen. Obviously, $u_2 + v_2 = D$. Using the micro meter, the distance between the two slits (d_2) is measured.

By definition of magnification, $\frac{d_1}{d} = -\frac{v_1}{u_1}$ and $\frac{d_2}{d} = -\frac{v_2}{u_2}$.

But in conjuagte foci method, $v_1 = u_2$ and $v_2 = u_1$.

$$\text{So, } \frac{v_1}{u_1} \cdot \frac{v_2}{u_2} = 1$$

$$\text{Then, } \frac{d_1}{d}\frac{d_2}{d} = 1 \quad \text{or} \quad \frac{d_1\, d_2}{d^2} = 1 \quad \text{or} \quad d^2 = d_1\, d_2 \quad \text{or} \quad d = \sqrt{d_1\, d_2}$$

But, we know that, $\beta = \frac{\lambda D}{d}$

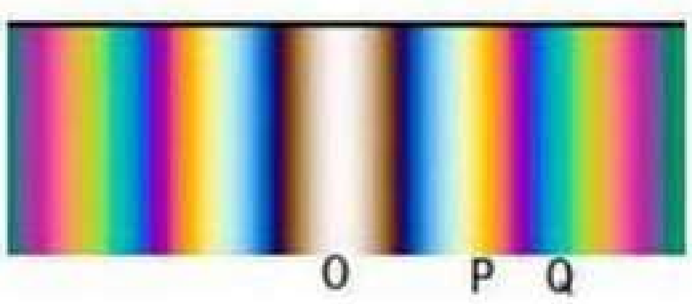
$$\text{Then, } \lambda = \frac{\beta\, d}{D}$$

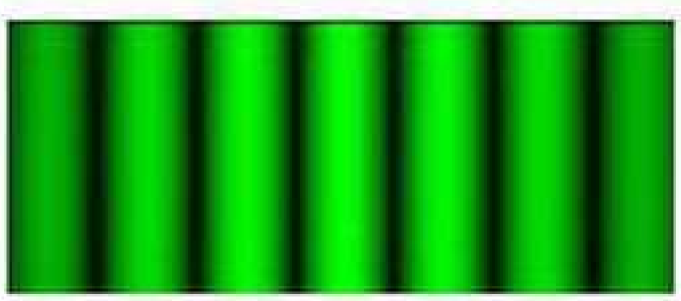
$$\lambda = \frac{\beta \sqrt{d_1\, d_2}}{D}$$

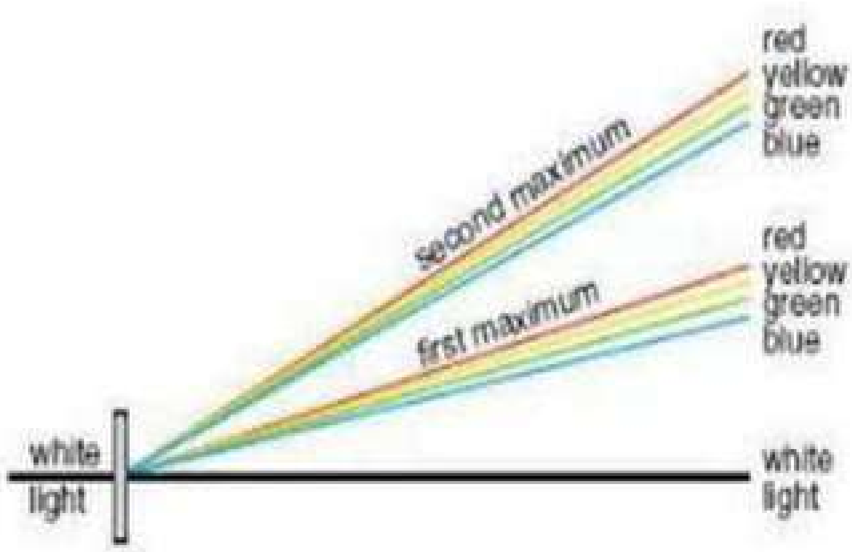
<u>Interference with white light:</u> [Can white light produce interference pattern? Answer: **Yes**]

A white light has a spectrum of colours from **violet** to **red**. The wavelength of violet is about $\mathbf{4 \times 10^{-5} cm}$ and that of red colour is $\mathbf{7 \times 10^{-5} cm}$ respectively.

The central bright fringe produced at O will be white, because, all wavelengths will constructively interfere here. Slightly away from O, the fringes at P and Q will be coloured.







In the usual interference pattern obtained with a monochromatic source (sodium vapour lamp), a large number of fringes with a single colour is seen and is extremely difficult to find the position of the central fringe. In case, if we need to find the central bright fringe, a while light is suitable than a monochromatic source.

Chapter 7: Interference of light by division of amplitude

Syllabus: Interference by a plane parallel thin film illuminated by a plane wave, Interference by a film with two non-parallel (Wedge) reflecting surfaces, colour of thin films, Theory of Newton's rings, Michelson Interferometer – Construction and Determination of wavelength of light, Numerical Problems. (05 hours)

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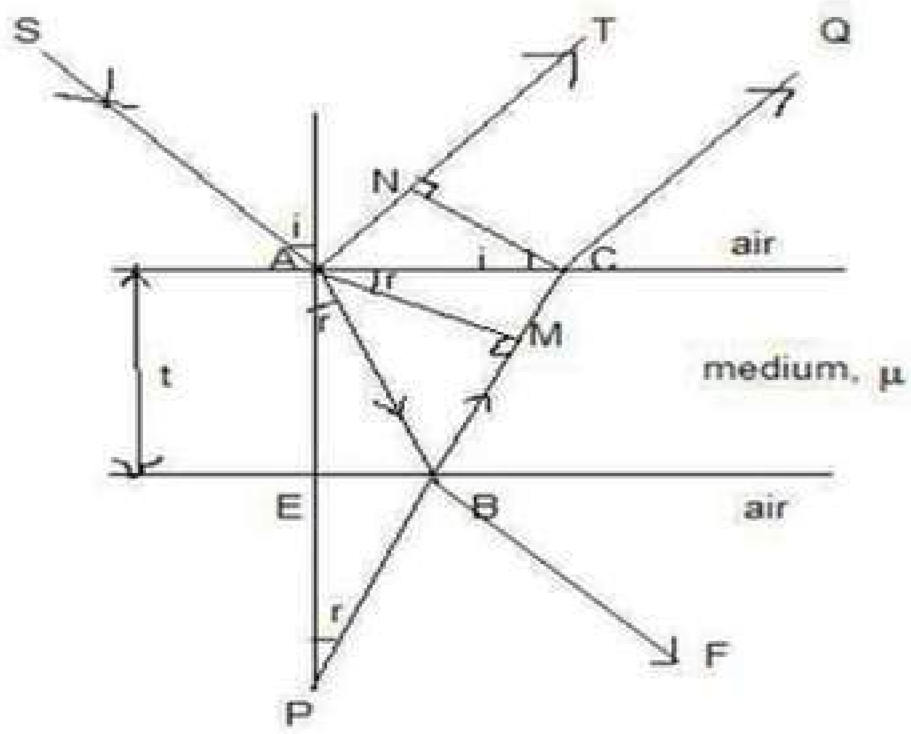
Division of Amplitude: In this method, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Thus we have coherent beams produced by division of amplitude. These beams travel different paths and are finally brought together to produce interference.

The experiments employed to obtain interference by division of amplitudes are interference due to thin films, wedge experiment, Newton's rings and Michelson's interferometer etc.

Interference by a plane parallel thin film illuminated by a plane wave:

When plane light waves fall on a thin film of uniform thickness, the plane waves get reflected by the top and bottom surfaces of a thin film and interfere with one another.



Consider a transparent thin film of thickness t and refractive index μ. A ray SA incident on the upper surface of a thin film of thickness t, is partially reflected along AT and partially refracted along AB. At B, a part of is again reflected along BC and the remaining light emerges out through BF (fig.)

The diffrence in path difference between the two rays AT and CQ can be found. Draw CN normal to AT and draw AM normal to BC. Let i be the angle of incidence and r the angle of referection. Produce CB to meet AE produced, at P. Hence, $\mathbf{A\hat{P}C = r}$.

The optical path difference, $x = \mu\,(AB + BC) - AN$ .... (1)

But, from Snell's law, $\mu = \frac{\sin i}{\sin r} = \frac{\left(\frac{AN}{AC}\right)}{\left(\frac{CM}{AC}\right)} = \frac{AN}{CM}$

or $AN = \mu\,(CM)$

Then, eqn.(1) becomes,

optical path difference, $x = \mu\,(AB + BC) - \mu\,(CM)$

$= \mu\,(AB + BC) - \mu\,(CM)$

$= \mu\,(AB + BC - CM)$

$= \mu\,(PB + BM)$ since **AB = PB**

optical path difference, $x = \mu\,(PM)$ .... (2)

From Δ^{le} APM, $\cos r = \frac{PM}{AP}$

or $PM = AP\cos r = 2t\cos r$ **since AP = 2t**

Then, eqn.(2) becomes,

optical path difference, $x = 2\,\mu\,t\cos r$ ....(3)

But, when light gets reflected by an optically denser medium, a phase change of π, **equivalent to a path difference of** $\frac{\lambda}{2}$ occurs. So, a **correction is needed** to the above optical path difference.

Therefore, the **corrected optical path diffrence,** $x = 2\,\mu\,t\cos r - \frac{\lambda}{2}$ .... (4)

i) **Condition for Maxima:** That is **constructive interference** to take place (bright vision).

the path diffrence, $x = n\lambda$, **where n = 0, 1, 2, ...**

Then, from **eqn.(3),**

$$2\,\mu\,t\cos r - \frac{\lambda}{2} = n\lambda$$

$$2\,\mu\,t\cos r = n\lambda + \frac{\lambda}{2}$$

$$2\,\mu\,t\cos r = (2n+1)\frac{\lambda}{2}$$

where n = 0, 1, 2, ...

ii) **Condition for Minima:** That is **destructive interference to take place** (dark vision).

the path diffrence, $x = (2n+1)\,\frac{\lambda}{2}$, **where n = 0, 1, 2, ...**

Then, from **eqn.(3),**

$$2\,\mu\,t\cos r - \frac{\lambda}{2} = (2n+1)\,\frac{\lambda}{2}$$

$$2\,\mu\,t\cos r = (2n+1)\,\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\,\mu\,t\cos r = (n+1)\,\lambda$$

Here, **n** is an integer and so **(n +1)** is also an integer and can be replaced by **'n'** itself.

Therefore, $2\,\mu\,t\cos r = n\lambda$ **where n = 0, 1, 2, ...**

It is very clear that the path difference depends upon the thickness 't' of the thin film and the angle of refraction 'r', i.e., actually, depends on the angle of incidence 'i'.

Colour of thin films:

When a thin film is illuminated by white light, the film shows **different colours**. The reason is given below:

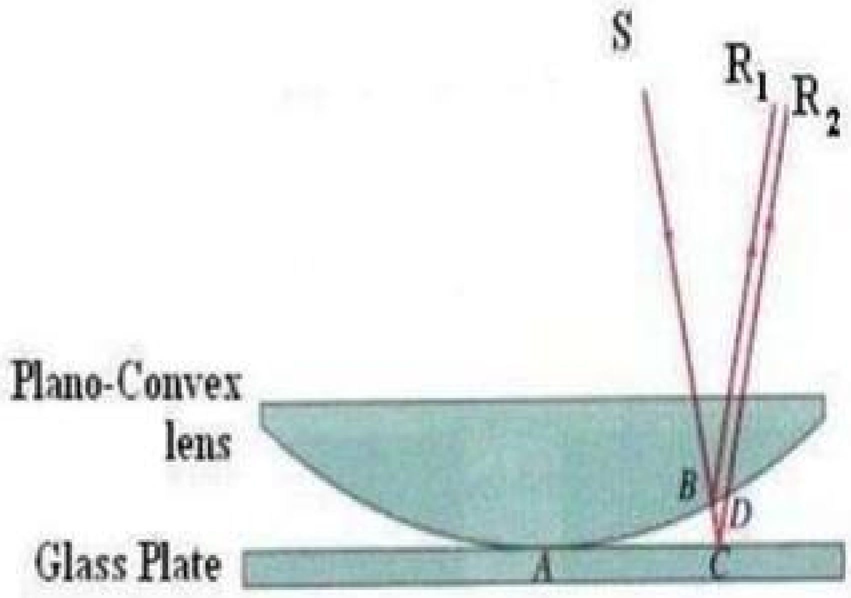
The incident wave of light is reflected from the upper and lower surfaces of the thin film. These two reflected rays are very close to each other and interfere. The path difference (for maxima) between the interfering waves is $2\,\mu\,t\cos r - \frac{\lambda}{2}$.

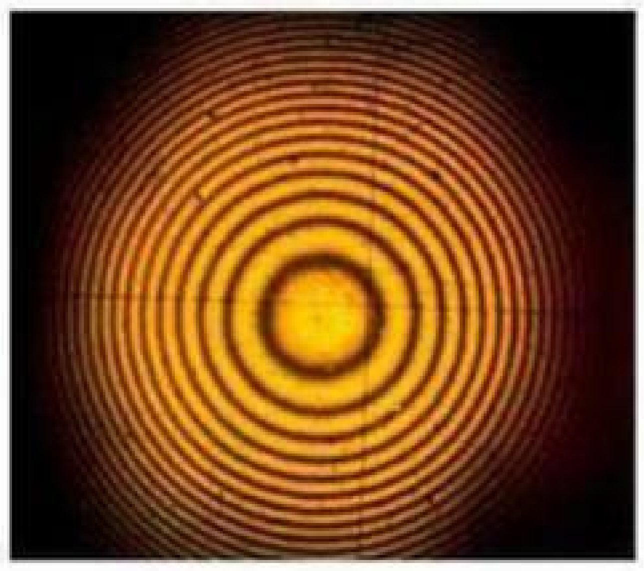
Now, white light consists of continuous range of wavelengths (colours). For a particular value of 't' and 'r', the waves of **only certain wavelengths** (colours) satisfy the **condition of maxima**, i.e., $2\,\mu\,t\cos r = (2n+1)\frac{\lambda}{2}$. Therefore, only those colours will be present in the reflected waves with maximum intensity.

The other nearby colours will be present with lesser intensity. At the same times, some colours may satisfy the condition for minima, i.e., $2\,\mu\,t\cos r = n\lambda$ and these colours will be missing from the reflected light.

Newton's Rings:

Newton's ring is a phenomenon in which an interference pattern is created by the reflection of light between two surfaces, namely, a spherical surface (plano-convex) and an adjacent touching glass plate. (fig.) R_1 *and* R_2 are the incident rays and S represents the reflected rays producing interference pattern.





When viewed with monochromatic light, Newton's rings appear as a series of concentric, alternating bright and dark rings, centered at the point of contact between the two surfaces. (fig.)

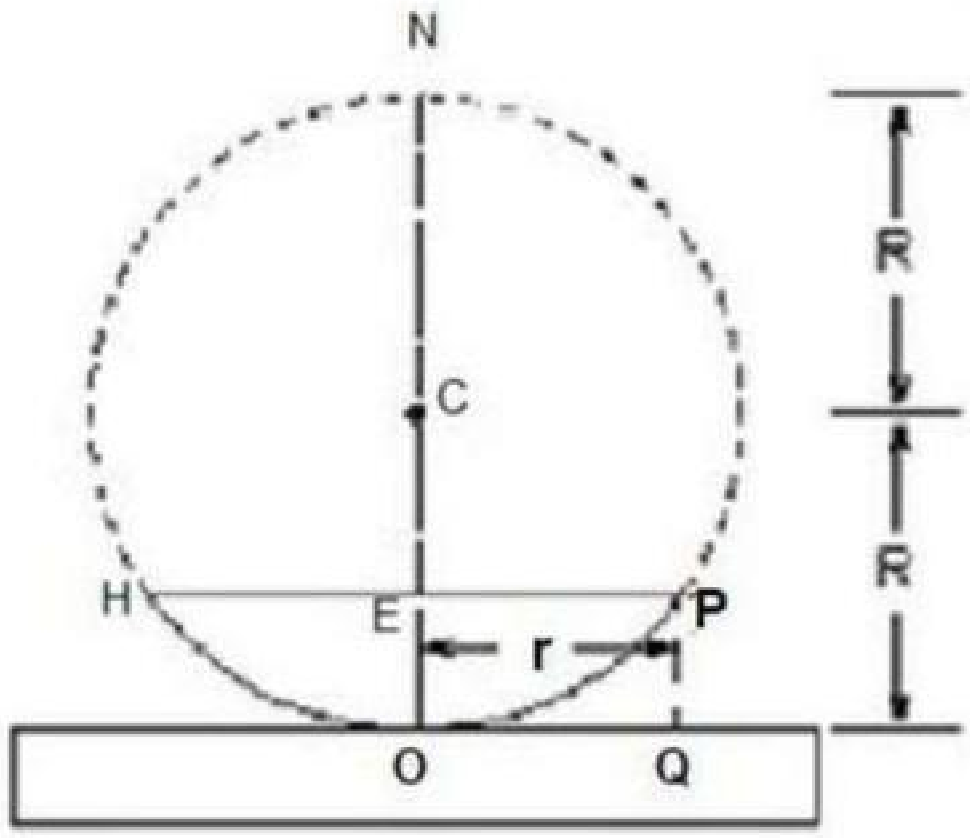
Theory of Newton's Rings:

Let 'R' be the radius of curvature of the plano-convex lens used, 't' the thickness of the air film at a distance 'r' from the centre, i.e., **OQ = r**. Here too, the interference takes place due to reflection of light. Then the path difference for **dark fringes** is given by,

$$2\,\mu\, t \cos r = n\lambda, \quad \text{where } n = 0, 1, 2, \ldots$$

Here, θ is very small and $\cos\theta = 1$. For air, $\mu = 1$. Then,

$$2\,t = n\lambda, \quad \text{where } n = 0, 1, 2, \ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$



From fig.,

$$HE \times EP = OE \times EN$$
$$HE \times EP = OE \times (2R - OE) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But, $EP = EH = r$ and $OE = PQ = t$

Since, $2R >> t$, then, $(2R - OE) = (2R - t) \cong 2R$

Then, from eqn.(2), $r^2 = 2Rt$ or $2t = \dfrac{r^2}{R}$

Substituting in eqn.(1), we get, $n\lambda = \dfrac{r^2}{R}$

$$r^2 = n\lambda R$$

$$\text{or} \quad r_n = \sqrt{n\lambda R} \qquad \text{where } n = 0, 1, 2, \ldots$$

Then, the diameter of the dark rings becomes, $D_n = 2r_n = 2\sqrt{n\lambda R}$

i.e., $D_n \,\alpha\, \sqrt{n}$

i.e., the diameters of the successive dark rings are proportional to the square roots of natural numbers.

When $n = 0, D = 0$. This corresponds to the centre of the Newton's rings. Thus, while counting the order of the dark rings, the central ring is not counted.

Note: The Newton's rings are circular. Why? **Answer:** The air film between the concave surface of the lens and the glass plate is circular with its centre at the point of contact. Hence, the fringes are **circular rings** with the common centre at the point of contact.

Note: In case of 4th and 9th dark rings, $D_4 = 2\sqrt{4\lambda R} = 4\sqrt{\lambda R}$ and $D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$

The difference between 4th and 9th dark rings, $D_9 - D_4 = 6\sqrt{\lambda R} - 4\sqrt{\lambda R} = 2\sqrt{\lambda R}$

Similarly, in case of 9th and 16th dark rings, $D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$ and $D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R}$

The difference between 9th and 16th dark rings, $D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R}$

It is clear that the fringe width decreases with the order of the fringes, i.e., the fringes get closer with increase in their order.

Measuring wavelength (λ) of the Sodium light using Newton's rings:

We know that, the diameter of the nth **dark ring**, $D_n = 2\sqrt{n\lambda R}$

or $D^2_n = 4n\lambda R$

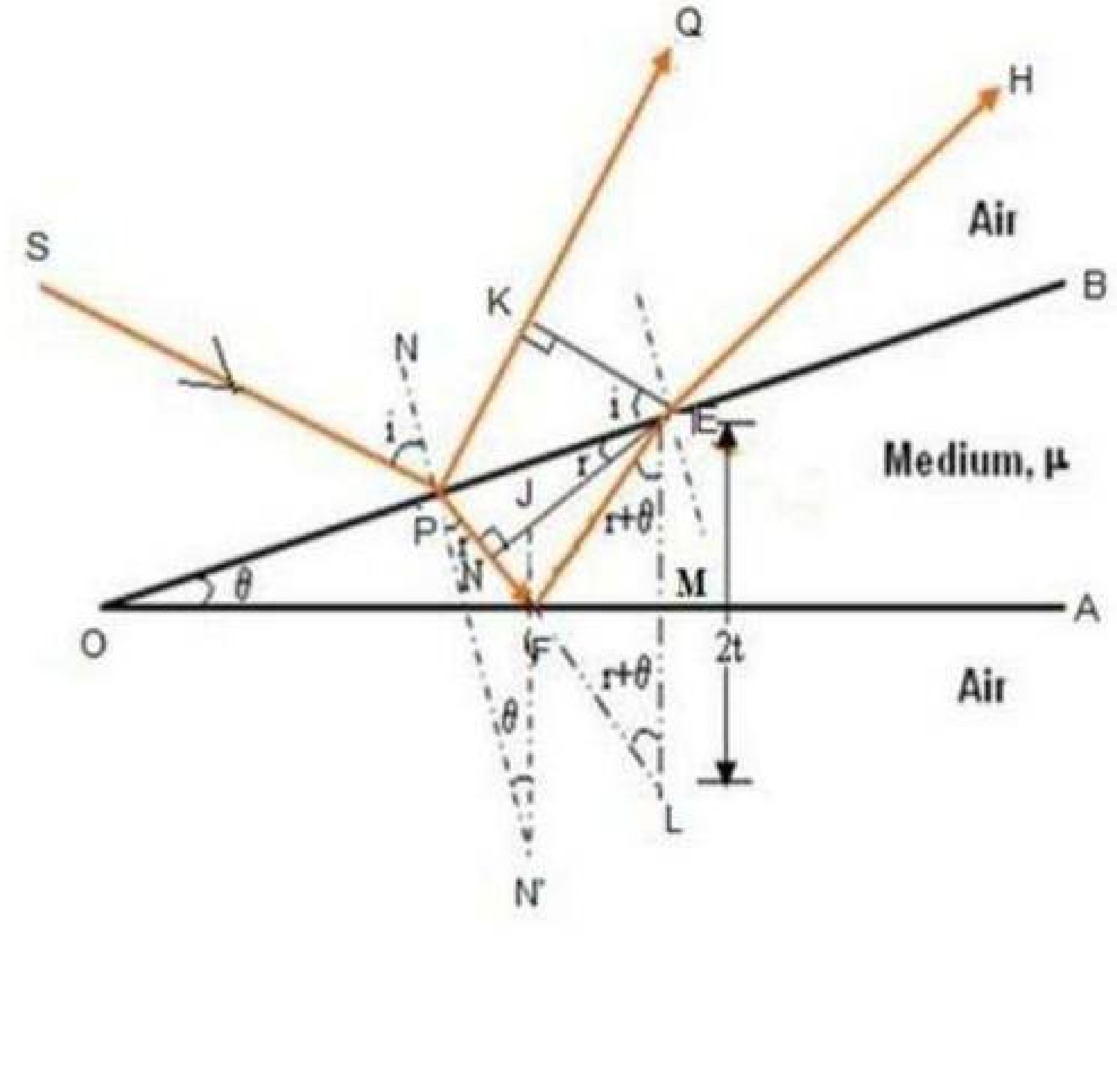
Similarly, the diameter of the mth **dark ring**, $D^2_m = 4m\lambda R$

Then, $D^2_m - D^2_n = 4\lambda R(m - n)$

$$\text{or} \quad \lambda = \frac{D_m^2 - D_n^2}{4\,R\,(m-n)}$$

Interference by a film with two non-parallel reflecting surfaces **(OR)**

Interference at a wedge: A wedge can be formed by introducing a piece of paper or a thin wire at one end of the combination of two identical glass plates placed one over the other.



When a **wedge shaped** thin film of some transparent material is exposed to light, due to interference of reflected rays, interference patterns are seen. This happens by the division of amplitudes (figure).

Let SP be the incident ray, PQ and EH be the interfering reflected rays from the two plane surfaces OA and OB respectively. To evaluate the optical path difference between these two rays, EK perpendicular to PQ and EN perpendicular to PF are drawn.

The **optical path difference** between the rays PQ and EH is,

$$\Delta = \mu\,(PF + FE) - PK \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From Snell's law, $\mu = \frac{\sin i}{\sin r} = \frac{PK/PE}{PN/PE} = \frac{PK}{PN}$

or $PK = \mu\,(PN)$

Then, eqn.(1) becomes,

optical path difference, $\Delta = \mu\,(PN + NF + FE) - \mu\,(PN)$

$$\Delta = \mu\,(PN + NF + FE - PN)$$

$$\Delta = \mu\,(NF + FE) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Now, draw EM perpendicular to OA and produce PF to meet EM produced at L. Now the Δ^{les} **EFM** and LFM are **congruent**. Also,

EM = ML = t and **FE = FL**

Then, the **optical path difference,** $\Delta = \mu\,(NF + FL)$

$$\Delta = \mu\,(NL) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Let θ be the angle enclosed between the plane surfaces OA and OB. Then the two normal drawn to OA and OB at P and F respectively, must also form an angle θ between them. (fig.)

i.e., $P\hat{F}J = r + \theta = E\hat{L}F$

From Δ^{le} **NEL**, $\cos(r+\theta) = \frac{NL}{EL}$

$$NL = EL\cos(r+\theta) = 2\,t\cos(r+\theta)$$

Eqn.(3) becomes, **the optical path difference,** $\Delta = 2\,\mu\,t\cos(r+\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$

But, when light gets reflected by an optically denser medium, a phase change of π, equivalent to a **path difference of** $\frac{\lambda}{2}$ occurs. So, a **correction is needed** to the above optical path difference.

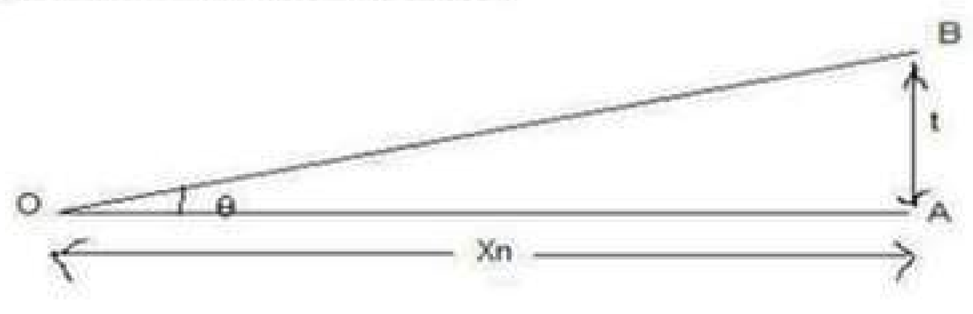
i) For constructive interference (bright fringes), the path difference is

$$2\,\mu\,t\cos(r+\theta) = (2n+1)\frac{\lambda}{2} \qquad \text{where } n = 0, 1, 2, \ldots$$

ii) For destructive interference (dark fringes), the path difference is

$$2\,\mu\,t\cos(r+\theta) = n\lambda \qquad \text{where } n = 0, 1, 2, \ldots$$

Expression for fringewidth using a wedge:



For n^{th} dark fringe, path difference, $2\,\mu\,t\cos(r+\theta) = n\lambda$ where n = 0, 1, 2, …

From fig., $\tan\theta = \frac{t}{x_n}$ or $t = x_n\tan\theta$

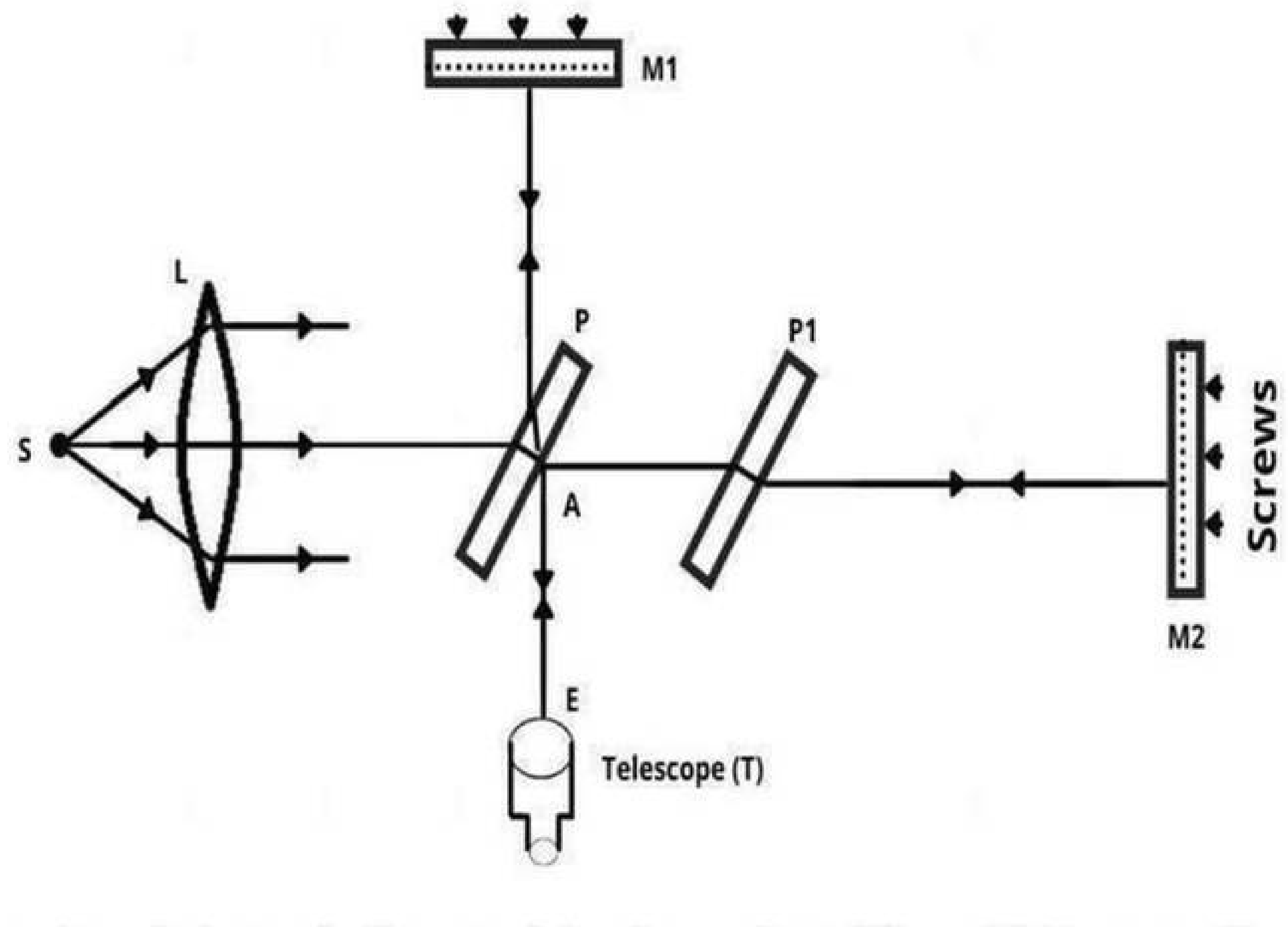
Then, $2\,\mu\,x_n\tan\theta\cos(r+\theta) = n\,\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$

Michelson Interferometer:

Construction: The Michelson interferometer is an accurate instrument. Figure depicts Michelson interferometer and the path of a light beam from a single point source S.

It essentially consists of a half-silvered mirror (beam splitter) P, with its surface inclined at an angle of 45° with the incident beam of light and an another glass plate P_1, such that P and P_1 are of the same thickness and same material and two plane mirrors M_1 and M_2 mounted vertically at right angles to each other. The planes of these mirrors can be slighted tilted with fine screws at their back (Figure). The mirror M_1 is movable and M_2 is fixed.

Working: Light from a monochromatic source **S** falls on the half-silvered mirror P. It splits the incident beam into two parts of nearly equal intensities, namely, reflected and transmitted beams.



The reflected beam travels towards the movable plane mirror M_1 and falls normally on it and hence it is reflected back to P and gets transmitted towards the telescope T. The transmitted beam moves towards the mirror M_2 and falls on it normally, after passing through a compensatory plate P_1. It is then reflected back by the mirror M_2 and **retraces** its path. At A on the plate P, it is reflected and enters the telescope T.

Since, both the beams entering the telescope are **coherent** in nature, they are capable of producing interference pattern.

Function of the compensating plate P_1: From the figure, it is noticed that the beam towards the mirror M_1 passes through P three times and the other towards M_2 only once. To ensure that both beams cross the same thickness of glass, a compensator transparent glass plate P_1 is placed in the arm containing M_2. This plate P_1 is duplicate of P, but **without silvering**.

Determination of wavelength (λ) of the light:

The mirrors M_1 and M_2 are adjusted at nearly equal distances from P and perpendicular to each other, so that circular fringes are obtained. The mirror M_1 is fixed and the position of the mirror M_2 is slightly adjusted to get a **bright spot** at the centre of the field of view.

For a given separation of 'd' between the positions of the mirror M_1 and M_2, the **path difference** is given by,

$$2d = n\lambda \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where '**n**' is the order of the spot obtained.

Now, if the mirror M_2 is moved by a small distance '**Δd**', so that **N** number of fringes cross the field of view, then the **path difference** is given by,

$$2(d + \Delta d) = (n + N)\lambda \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From eqns.(1) and (2), we get,

$$2(d + \Delta d) - 2d = (n + N)\lambda - n\lambda$$

$$2\,(\Delta d) = N\,\lambda$$

or

$$\lambda = \frac{2\,(\Delta d)}{N}$$

Thus, the value of the wavelength (**λ**) can be determined**,** if we measure the value of Δd using the micrometer and the count the number N.